Chapter Nine

Section 9.1 Odd Answers

1. A: (25, 0) B: (25, 3π/2) C: (15, π/6)

3. A: (30, π/3) B: (20, 5π/6) C: (25, 5π/4)

5. Graph is given. Shape is almost rectangular.

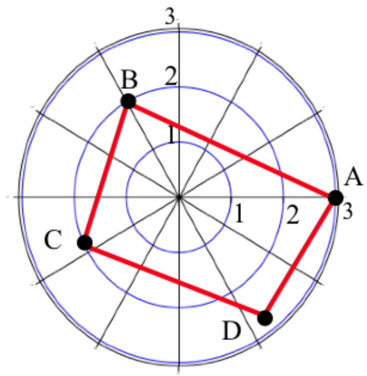
7. Graph is given.

9. Graph is given.

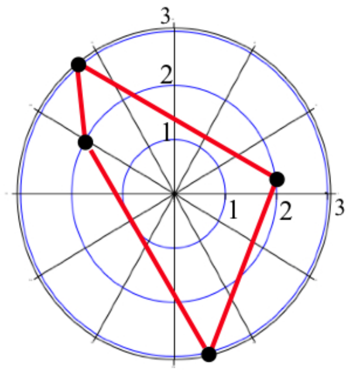
11. Graph is given.

13. Graph is given.

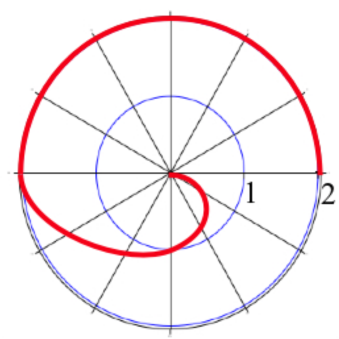
15. Graphs are given.



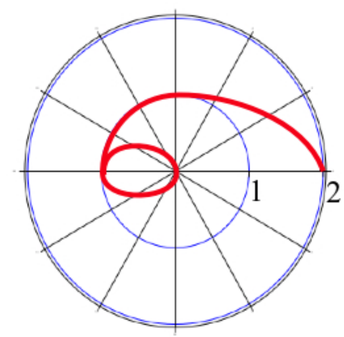
Prob. 5



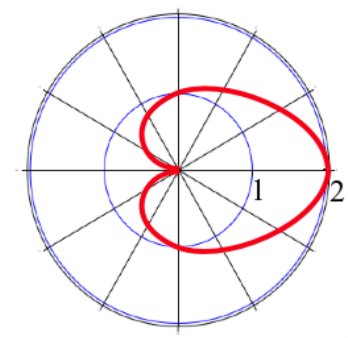
Prob. 7



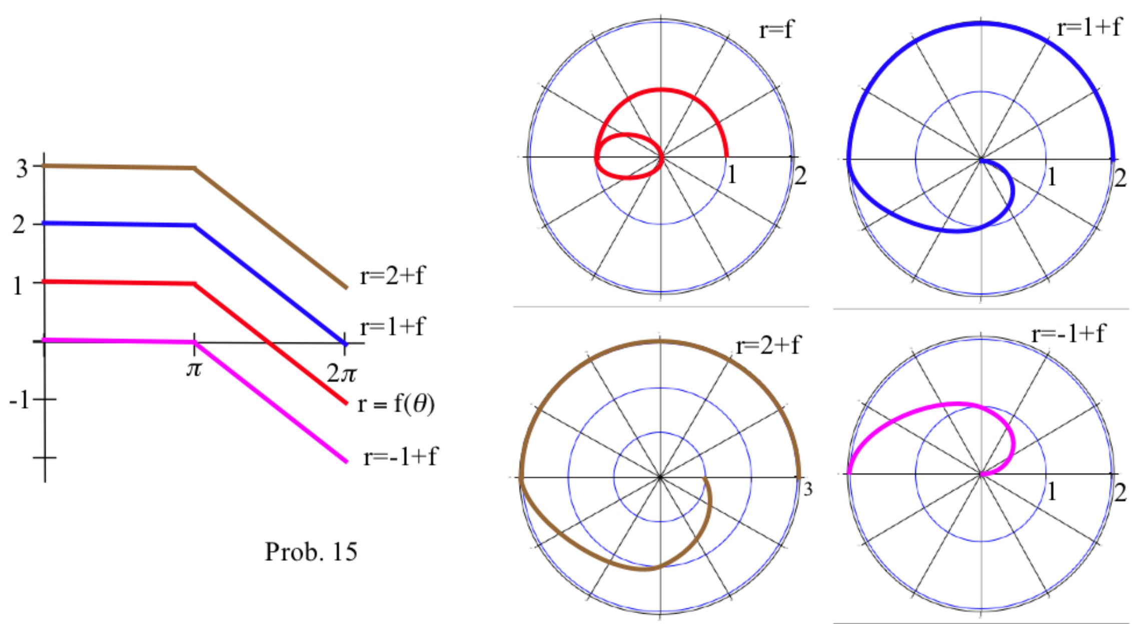
Prob. 9



Prob. 11

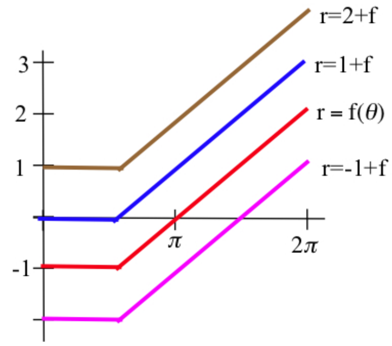


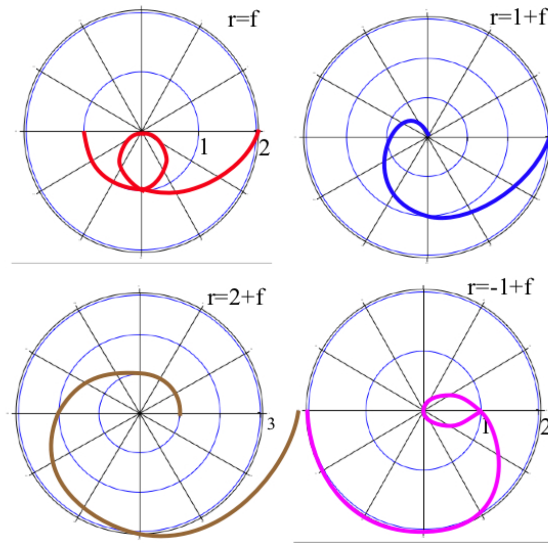
Prob. 13



Prob. 15

17. Graphs are given.



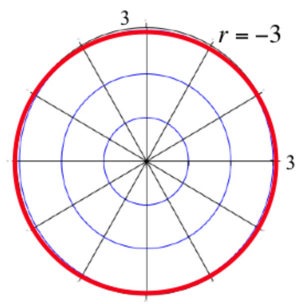


Prob. 17

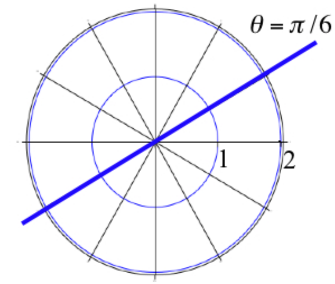
19. The polar graph will approach (spiral in or out to) a circle of radius 3.

21. Circle centered at origin with radius 3.

23. Line through the origin making an angle of π/6 with the x–axis.



Prob. 21



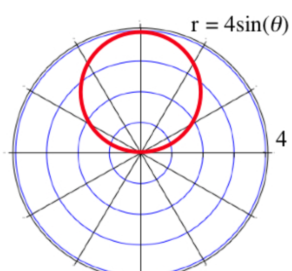
Prob. 23

25. A circle sitting atop the x–axis, touching the origin.

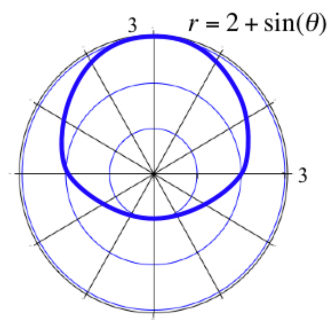
27. Graph is given.

29. Graph is given.

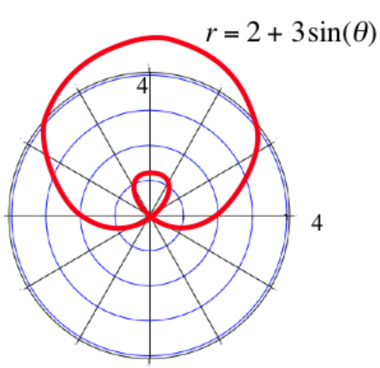
31. Graph is given.



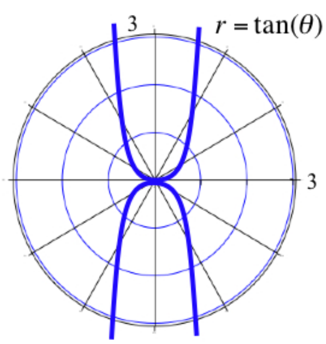
Prob. 25



Prob. 27

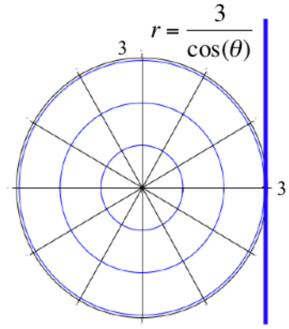


Prob. 29



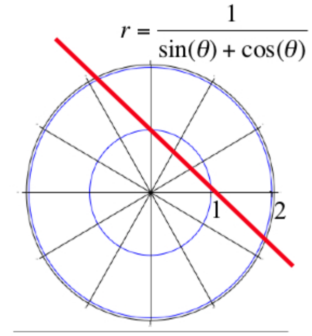
Prob. 31

33. Graph is given. A vertical line through the rectangular coordinate point (3,0).



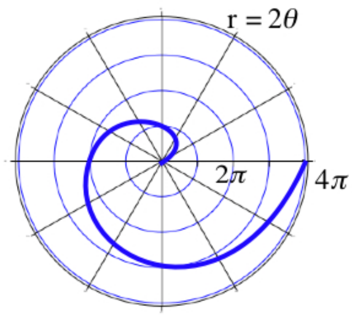
Prob. 33

35. Graph is given. A line: in rectangular coordinates $y = 1 - x$.



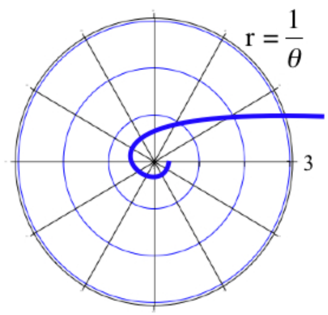
Prob. 35

37. Graph is given: a fast growing spiral.



Prob. 37

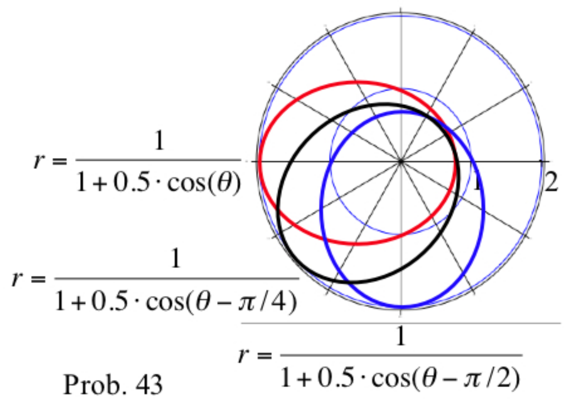
39. Graph is given.



Prob. 39

41. {m = 1, n = 2}, {m = 2, n = 4}, {m = 3, n = 3}, {m = 4, n = 4} all have pleasing shapes. Find some others for yourself.

43. The graphs are given. The graph for $a = 0$ is rotated $\pi/6$, $\pi/4$, and $\pi/2$ counterclockwise about the origin as $a = \pi/6$, $\pi/4$, and $\pi/2$ respectively.



Prob. 43

45.

n	1	2	3	4	5	6	7	8
# petals	1	4	3	8	5	12	7	16

47. Rectangular $(-2, 3)$ is polar $(\sqrt{13}, 2.159)$.
Rect $(2, -3)$ is polar $(\sqrt{13}, 5.300)$ or $(-\sqrt{13}, 2.159)$.
Rect $(0, -4)$ is polar $(-4, \pi/2)$ or $(4, 3\pi/2)$.

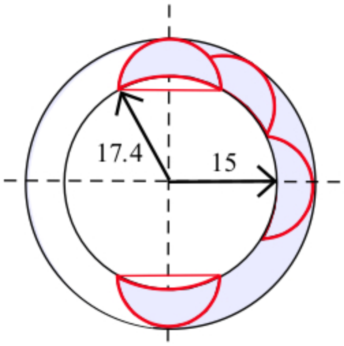
49. Rectangular $(3, 4)$ is polar $(5, 0.927)$.
Rect $(-1, -3)$ is polar $(\sqrt{10}, 4.391)$.
Rect $(-7, 12)$ is polar $(\sqrt{193}, -1.043)$ or $(\sqrt{193}, -1.043 + \pi) \approx (\sqrt{193}, 2.099)$.

51. Polar $(-2, 3)$ is rect. $(1.98, -0.282)$. Polar $(2, -3)$ is rect. $(-1.98, -0.282)$. Polar $(0, -4)$ is rect. $(0, 0)$.

53. Polar $(2, 3)$ is rect. $(-1.98, 0.282)$. Polar $(-2, -3)$ is rect. $(1.98, 0.282)$. Polar $(0, 4)$ is rect. $(0, 0)$.

55. $x = 15 \cdot \cos(-30^{\circ}) + 10 \cdot \cos(0^{\circ}) \approx 22.99$ inches to the right of the shoulder.
$y = 15 \cdot \sin(-30^{\circ}) + 10 \cdot \sin(0^{\circ}) \approx -7.5$ inches or 7.5 inches **below** the shoulder.
Polar location of hand: $(24.18, -18.07^{\circ})$ or $(24.18, -0.32)$.

57. $x = 15 \cdot \cos(-0.9) + 10 \cdot \cos(-0.5) \approx 18.10$ inches to the right of the shoulder.
$y = 15 \cdot \sin(-0.9) + 10 \cdot \sin(-0.5) \approx 16.54$ inches **below** the shoulder. Polar is $(24.52, -0.74)$.

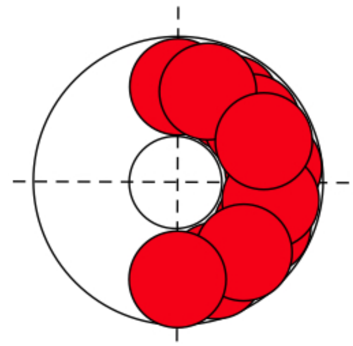


Prob. 59

59. Figure is given: $-\pi/2 \le \theta \le \pi/2$ and $-\pi/2 \le \phi \le \pi/2$. The robot's hand can reach the shaded region.

60. Figure is given: $-\pi/2 \le \theta \le \pi/2$ and $-\pi \le \phi \le \pi$. The robot's hand can reach the shaded region.

61. On your own.


Prob. 60

Section 9.2 Odd Answers

1.

Point	dr/dθ	dx/dθ	dy/dθ	dy/dx
A	–	–	+	–
B	0	–	+	–
C	+	–	+	–
D	+	–	0	0
E	–	+	–	–

2.

Point	dr/dθ	dx/dθ	dy/dθ	dy/dx
A	+	+	+	+
B	+	–	+	–
C	–	–	–	+
D	–	–	–	+
E	+	–	–	+

3.

Point	dr/dθ	dx/dθ	dy/dθ	dy/dx
A	0	0	+	Und.
B	+	0	+	Und.
C	U	U	U	U
D	–	–	0	0
E	+	–	0	0

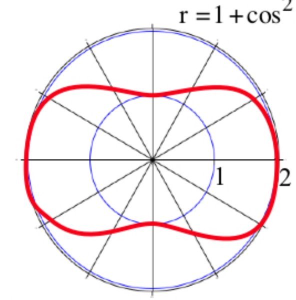
4.

Point	dr/dθ	dx/dθ	dy/dθ	dy/dx
A	–	+	–	–
B	–	+	–	–
C	+	+	–	–
D	+	+	0	0
E	+	+	+	+

5. The polar graph is a circle centered at the origin with radius 5.

At A: $dr/d\theta = 0$, $dy/dx = -1$. At B: $dr/d\theta = 0$, $dy/dx = 0$.

At C: $dr/d\theta = 0$, $dy/dx =$ Und.



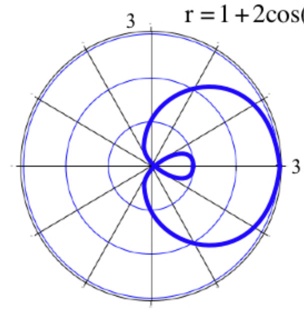
Prob. 7

7. The graph is given.

At A: $dr/d\theta = 0$, $dy/dx =$ Und. At B: $dr/d\theta = -1$, $dy/dx = -1/5$.

At C: $dr/d\theta = 0$, $dy/dx = 0$.

9. When $\theta = 2\pi/3$, $dy/dx = -\sqrt{3}$. When $\theta = 4\pi/3$, $dy/dx = +\sqrt{3}$.



Prob. 9

11. We have $$\frac{dy}{dx} = \frac{r\cdot\cos(\theta) + \frac{dr}{d\theta}\cdot\sin(\theta)}{-r\cdot\sin(\theta) + \frac{dr}{d\theta}\cdot\cos(\theta)} ,$$

so if $r(\theta) = 0$ (and $\frac{dr}{d\theta}$ exists and $\frac{dr}{d\theta} \ne 0$)

then $$\frac{dy}{dx} = \frac{\frac{dr}{d\theta}\cdot\sin(\theta)}{\frac{dr}{d\theta}\cdot\cos(\theta)} = \tan(\theta) .$$

13. $\frac{9}{16}\pi^2 \approx 5.552$

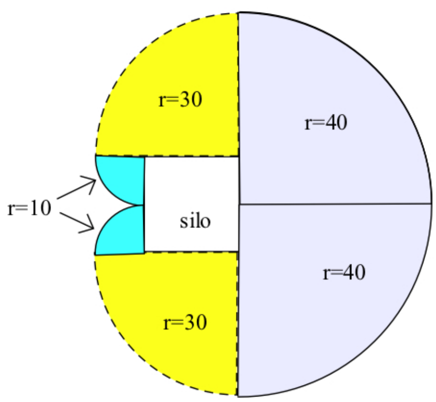
15. {area of cardioid in first quad.} – {area of circle in first quad.} $= \{ 1 + \frac{3\pi}{8} \} - \{ \frac{\pi}{4} \} = 1 + \frac{\pi}{8} \approx 1.393$.
This problem is worked out in Example 4.

17. 3–petal: $A = \frac{1}{2}\int_0^{\pi/3} \sin^2(3\theta)\, d\theta = \frac{\pi}{12} \approx 0.262$. 5–petal: $A = \frac{1}{2}\int_0^{\pi/5} \sin^2(5\theta)\, d\theta = \frac{\pi}{20} \approx 0.157$.

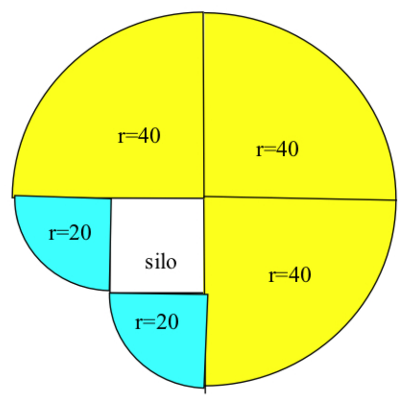
19. $A = \pi(2)^2 = 4\pi \approx 12.566$

21. (b) add several semicirular regions to get $\frac{1}{2}\pi(40)^2 + \frac{1}{2}\pi(30)^2 + \frac{1}{2}\pi(10)^2 = 1300\pi \text{ ft}^2 \approx 4{,}084.1 \text{ ft}^2$

(c) $\frac{3}{4}\pi(40)^2 + \frac{1}{2}\pi(20)^2 = 1400\pi \text{ ft}^2 \approx 4{,}398.2 \text{ ft}^2 > 4{,}084.1 \text{ ft}^2$



Prob. 21(a)



Prob. 21(b)

23. $r = \theta$, $dr/d\theta = 1$, $L = \int_0^{2\pi} \sqrt{\theta^2 + 1}\, d\theta \approx 21.256$ (using Simpson's rule with n = 20).

25. $r = 1 + \cos(\theta)$, $dr/d\theta = -\sin(\theta)$,

$$L = \int_0^{2\pi} \sqrt{\{1+\cos(\theta)\}^2 + \{-\sin(\theta)\}^2}\, d\theta = \int_0^{2\pi} \sqrt{2 + 2\cdot\cos(\theta)}\, d\theta = \int_0^{2\pi} \sqrt{4\cdot\cos^2(\theta/2)}\, d\theta$$

$$= 2\int_0^{2\pi} |\cos(\theta/2)|\, d\theta = 4\int_0^{\pi} \cos(\theta/2)\, d\theta = 8\sin(\theta/2)\Big|_0^{\pi} = 8 .$$

(Simpson's rule with n= 20 gives the same result.)

27. 10π

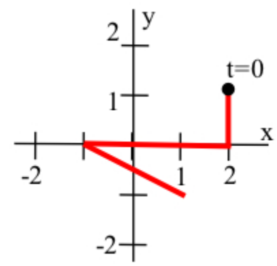
29. $r = \sin(3\theta)$, $dr/d\theta = 3\cdot\cos(3\theta)$, $L = \int_0^{\pi/3} \sqrt{\sin^2(3\theta) + \{3\cdot\cos(3\theta)\}^2}\, d\theta \approx 2.227$ (Simpson, n = 20).

Section 9.3 Odd Answers

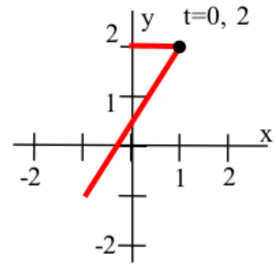
1. Graph is given. 3. Graph is given.

5. Graph is given. 7. Graph is given.

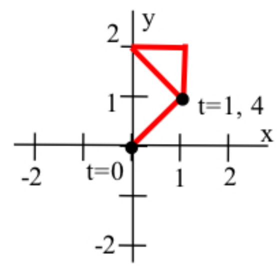
9. Graph is given: a straight line.



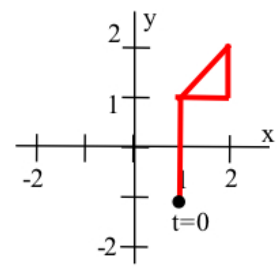
Prob. 1



Prob. 3



Prob. 5



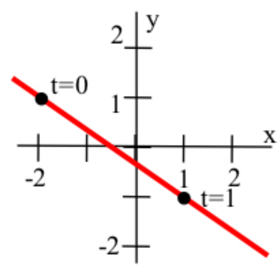
Prob. 7

11. (x(0), y(0)) = (b, d) and

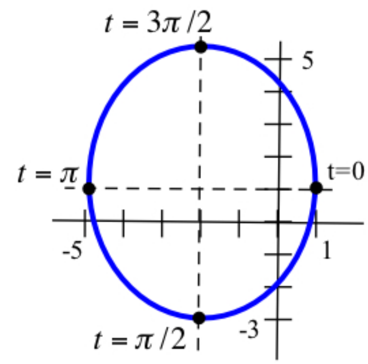
(x(1), y(1)) = (a+b, c+d) so

slope $= \dfrac{(c+d) - d}{(a+b) - b} = \dfrac{c}{a}$.

13. Graph is given.



Prob. 9



Prob. 13

15. Graphs are given.

(a) is the entire line y = x.

(b) is the "half–line" y = x for x ≥ 0.

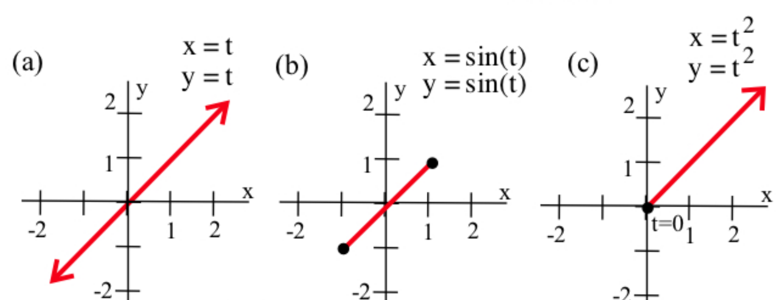
(c) is the line segment y = x for

–1 ≤ x ≤ 1: the location oscillates

along the line between (–1, –1) and (1,1).

All of these graphs satisfy the same relationship between x and y, y = x, but the graphs cover different parts of the graph of y = x (different domains).

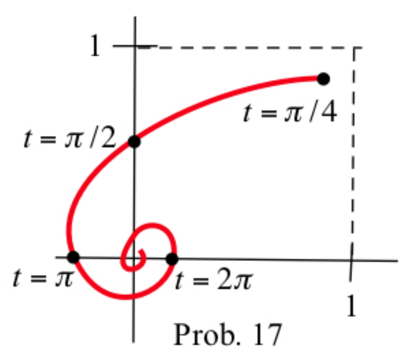
17. The graph is given. The graph begins at $\left(\frac{2\sqrt{2}}{\pi} , \frac{2\sqrt{2}}{\pi} \right)$ when t = π/4, and then it spirals counterclockwise around and in toward the origin. As t increases from π/4 , the radial distance of a point on the graph decreases, approaching 0.



Prob. 15

19. closer to

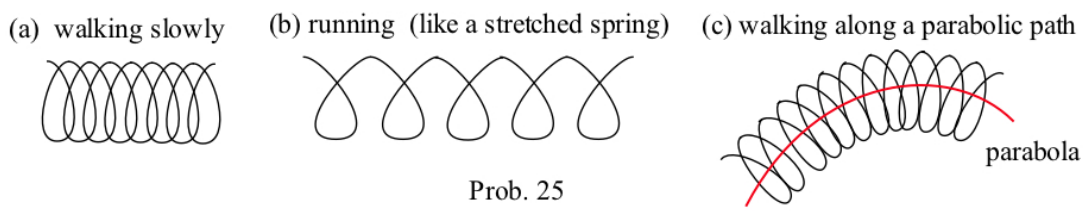
21. No. The new values give a point very close to the fixed point for these populations so the system will be in good balance.



Prob. 17

23. Solve $x = at + b$ for t and substitute this value of t into the equation $y = ct + d$. Then $t = \frac{x-b}{a}$ and $y = c\left(\frac{x-b}{a}\right) + d = \frac{c}{a}x + \left(d - \frac{bc}{a}\right)$, the equation of a straight line with slope $\frac{c}{a}$. The slope $\frac{c}{a}$ of y as a function of x is the slope of y as a function of t divided by the slope of x as a function of t.

25. Sketches of some possible paths are given.



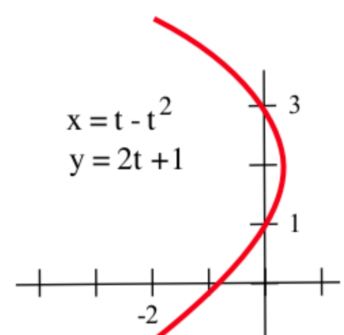
Prob. 25

27. (a) $x = R(t - \sin(t))$, $y = R(1 - \cos(t))$. If $t = 2\pi$, then $(x, y) = (2\pi R, 0) = (10\pi, 0)$ so $R = 5$.
    (b) Set $x = R(t - \sin(t)) = 5$ and $y = R(1 - \cos(t)) = 2$, divide y by x to eliminate R, solve (graphically or using Newton's method or some other way) to get $t \approx 3.820$. Substitute this value into the equation for x or y and solve for $R \approx 1.124$.
    (c) Set $x = R(t - \sin(t)) = 2$ and $y = R(1 - \cos(t)) = 3$, divide y by x to eliminate R, solve (graphically or using Newton's method or some other way) to get $t \approx 1.786$. Substitute this value into the equation for x or y and solve for $R \approx 2.472$.
    (d) Set $x = R(t - \sin(t)) = 4\pi$ and $y = R(1 - \cos(t)) = 8$ and solve to get $t = \pi$ and $R = 4$.

29. (a) $x = -50 + 30(\frac{1}{\sqrt{2}})\cdot t = -50 + (15\sqrt{2})\cdot t$ feet and
    $y = 6 + 30(\frac{1}{\sqrt{2}})\cdot t - 16t^2 = 6 + (15\sqrt{2})\cdot t - 16t^2$ feet.
    (b) $x = -50 + V(\frac{\sqrt{2}}{2})\cdot t$ feet and $y = 6 + V(\frac{\sqrt{2}}{2})\cdot t - 16t^2$ feet.

Section 9.4 Odd Answers

1. (a) The graph is given.
   (b) $dx/dt = 1 - 2t$, $dy/dt = 2$. When $t = 0$, $dy/dx = 2$. When $t = 1$, $dy/dx = -2$. When $t = 2$, $dy/dx = -2/3$.
   (c) dy/dx is never 0. dy/dx is undefined when $t = 1/2$: at $(x, y) = (1/4, 2)$.

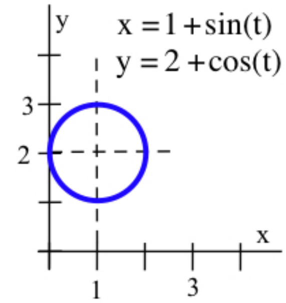


Prob. 1

3. (a) The graph is given.

   (b) $dx/dt = -\sin(t)$, $dy/dt = \cos(t)$. When $t = 0$, dy/dx is undefined. When $t = \pi/4$, $dy/dx = -1$. When $t = \pi/2$, $dy/dx = 0$.

   (c) $dy/dx = 0$ whenever $t = (k+\frac{1}{2})\pi$ for k an integer. dy/dx is undefined whenever $t = k\pi$ for k an integer.

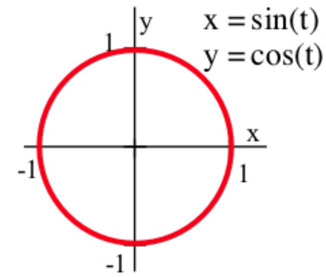


Prob. 3

5. (a) The graph is given.

   (b) $dx/dt = \cos(t)$, $dy/dt = -\sin(t)$. When $t = 0$, $dy/dx = 0$. When $t = \pi/4$, $dy/dx = -1$. When $t = \pi/2$, dy/dx is undefined. When $t = 17.3$, $dy/dx \approx 47.073$.

   (c) $dy/dx = 0$ whenever $t = k\pi$ for k an integer. dy/dx is undefined whenever $t = (k + \frac{1}{2})\pi$ for k an integer.

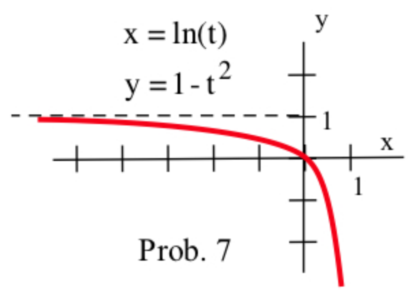


Prob. 5

7. (a) The graph is given.

   (b) $dx/dt = 1/t$, $dy/dt = -2t$. When $t = 1$, $dy/dx = -2$. When $t = 2$, $dy/dx = -8$. When $t = e$, $dy/dx = -2e^2$.

   (c) The function is only defined for $t > 0$, and for all $t > 0$ the slope of the tangent line dy/dx is defined and is not equal to 0.



Prob. 7

9. (a) $m_0 \approx \frac{-1}{1} = -1$. $m_1 \approx \frac{0}{1} = 0$. $m_2 \approx \frac{0}{1} = 0$. m_3 is undefined.

   (b) $dy/dx = 0$ when $t = 1$ and $t = 2$.

11. (a) $m_0 \approx \frac{2}{-1} = -2$. $m_1 \approx \frac{1}{-1} = -1$. m_2 is undefined. $m_3 = \frac{-1}{1} = -1$.

   (b) dy/dx is undefined when $t = 2$.

13. $dx/dt = 1 - 2t$, $dy/dt = 2$ so $v = \sqrt{(1-2t)^2 + (2)^2} = \sqrt{4t^2 - 4t + 5}$.
$v_0 = \sqrt{5} \approx 2.24$ ft/s, $v_1 = \sqrt{5} \approx 2.24$ ft/s, $v_2 = \sqrt{13} \approx 3.61$ ft/s.

15. $dx/dt = -\sin(t)$, $dy/dt = \cos(t)$ so
$v = \sqrt{(-\sin(t))^2 + (\cos(t))^2} = \sqrt{\sin^2(t) + \cos^2(t)} = 1$ ft/s for all values of t.
$v_0 = v_{\pi/4} = v_{\pi/2} = v_\pi = 1$ ft/s.

17. $v_0 \approx \sqrt{(1)^2 + (-1)^2} = \sqrt{2} \approx 1.41$ ft/s. $v_1 \approx \sqrt{(1)^2 + (0)^2} = 1$ ft/s.
$v_2 \approx \sqrt{(1)^2 + (0)^2} = 1$ ft/s. $v_3 \approx \sqrt{(0)^2 + (-1)^2} = 1$ ft/s.
$v_4 \approx \sqrt{(-1)^2 + (-1)^2} = \sqrt{2} \approx 1.41$ ft/s.

19. $v_0 \approx \sqrt{(-1)^2 + (2)^2} = \sqrt{5} \approx 2.24$ ft/s. $v_1 \approx \sqrt{(-1)^2 + (1)^2} = \sqrt{2} \approx 1.41$ ft/s.
$v_2 \approx \sqrt{(0)^2 + (0)^2} = 0$ ft/s. $v_3 \approx \sqrt{(1)^2 + (-1)^2} = \sqrt{2} \approx 1.41$ ft/s.

21. (a) $dx/dt = R(1 - \cos(t))$ and $dy/dt = R\sin(t)$ so
$v = \sqrt{R^2(1 - \cos(t))^2 + R^2(\sin(t))^2} = |R|\sqrt{2}\sqrt{1 - \cos(t)}$ ft/s.
(b) v is maximum when $\cos(t) = -1$, when $t = (2k + 1)\pi$ seconds for k an integer.
(c) $v_{max} = 2R$ ft/s.

23. $L = \int_0^2 \sqrt{(1 - 2t)^2 + 2^2}\, dt \approx 4.939$ (using a calculator).

25. π (half the circumference of the circle).

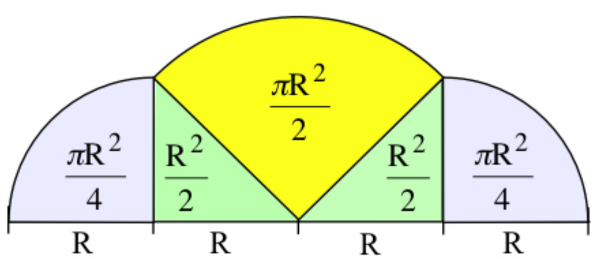
27. $x = 3 - t, y = 1 + \frac{1}{2} t$. $L = \int_1^3 \sqrt{(-1)^2 + (1/2)^2}\, dt = 2 \cdot \frac{\sqrt{5}}{2} = \sqrt{5} \approx 2.24$.

Alternately, the graph of (x, y) is a straight line, and we can calculate the distance from (x(1), y(1)) = (2,1.5) to the point (x(3), y(3)) = (0,2.5): distance = $\sqrt{2^2 + 1^2} = \sqrt{5}$.

29. $y = 4t^2 - t^4, dx/dt = 2t$. $A = \int_0^2 (4t^2 - t^4) \cdot 2t\, dt = 2t^4 - \frac{1}{3} t^6 \Big|_0^2 = 32 - \frac{64}{3} = \frac{32}{3}$.

31. $y = 1 + \cos(t), dx/dt = 2t$. $A = \int_0^2 (1 + \cos(t)) \cdot 2t\, dt \approx 4.805$ (using a calculator).

33. See Fig. 24. $A = \frac{\pi R^2}{4} + \frac{R^2}{2} + \frac{\pi R^2}{2} + \frac{R^2}{2} + \frac{\pi R^2}{4} = \pi R^2 + R^2 = R^2(\pi + 1)$.



Prob. 33

Section 9.4.5 Odd Answers

1. $P_A = (0,4)$, $P_B = (5,2)$. $x(t) = 0 + 5t = (1-t)\cdot 0 + t\cdot 5$ and $y(t) = 4 + (-2)t = (1-t)\cdot 4 + t\cdot 2$.

3. $P_A = (4,3)$, $P_B = (1,2)$. $x(t) = 4 - 3t = (1-t)\cdot 4 + t\cdot 1$ and $y(t) = 3 - 1t = (1-t)\cdot 3 + t\cdot 2$.

5. $P_A = (1,4)$, $P_B = (5,1)$. $x(t) = 1 + 4t = (1-t)\cdot 1 + t\cdot 5$ and $y(t) = 4 + (-3)t = (1-t)\cdot 4 + t\cdot 1$

7. If we start with the equation $x(t) = x_0 + t\cdot\Delta x$ and replace Δx with $x_1 - x_0$ then $x(t) = x_0 + t\cdot(x_1 - x_0) = x_0 + t\cdot x_1 - t\cdot x_0 = (1-t)\cdot x_0 + t\cdot x_1$ which is the pattern we wanted. The algebra for y(t) is similar.

For problems 9-13 the Bezier pattern is

$$B(t) = (1-t)^3\cdot P_0 + 3(1-t)^2 t\cdot P_1 + 3(1-t)t^2\cdot P_2 + t^3\cdot P_3$$

9. $x(t) = (1-t)^3\cdot 0 + 3(1-t)^2 t\cdot 2 + 3(1-t)t^2\cdot 1 + t^3\cdot 4$
$y(t) = (1-t)^3\cdot 5 + 3(1-t)^2 t\cdot 3 + 3(1-t)t^2\cdot 4 + t^3\cdot 2$

11. $x(t) = (1-t)^3\cdot 6 + 3(1-t)^2 t\cdot 6 + 3(1-t)t^2\cdot 2 + t^3\cdot 2$
$y(t) = (1-t)^3\cdot 5 + 3(1-t)^2 t\cdot 3 + 3(1-t)t^2\cdot 5 + t^3\cdot 0$

13. $P_0 = (5,1)$ and $B'(0) = 2$ tells us that P_1 could be (5+1, 1+2) or (5+2, 1+4) or (5+h, 1 + 2h)
We pick $P_1 = (6,3)$
$P_3 = (1,3)$ and $B'(1) = 3$ tells us P_2 could be (1+1,3+3) or (1+h, 3+3h).
We pick $P_2 = (2,6)$.

$$x(t) = (1-t)^3\cdot 5 + 3(1-t)^2 t\cdot 6 + 3(1-t)t^2\cdot 2 + t^3\cdot 1$$
$$y(t) = (1-t)^3\cdot 1 + 3(1-t)^2 t\cdot 3 + 3(1-t)t^2\cdot 6 + t^3\cdot 3$$

14. See Fig. 20. 15. See Fig. 21.

16. See Fig. 22. 17. See Fig. 23.

18. Violates Property (3): B'(1) does not equal the slope of the segment from P_2 to P_3

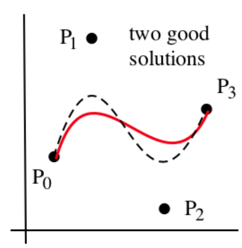


Fig. 20

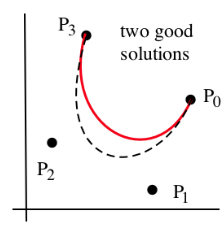


Fig. 21

19. Violates Property (3) at B(1). Also violates Property (4) since the B(t) graph goes outside the "rubber band" around the 4 control points.

20. Violates Property (2). Since the graph of B(t) has 3 "turns" then B(t) is not a cubic polynomial (which can only have 2 turns).

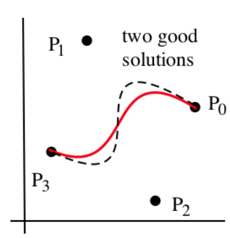


Fig. 22

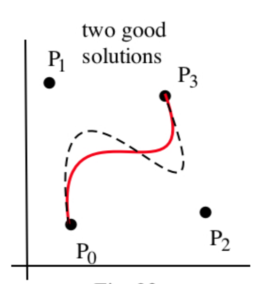
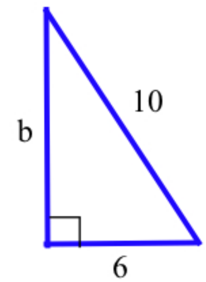


Fig. 23

21. Violates Property (1): B(1) = P_2 instead of P_3.

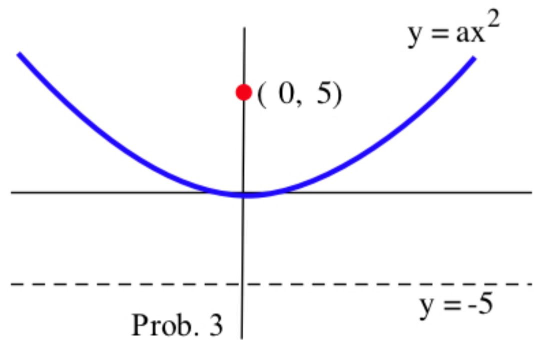
Section 9.5 Odd Answers



Prob. 1

1. An ellipse. Figure is given Since $b^2 + 6^2 = 10^2$, $b = 8$. $c = \sqrt{a^2 - b^2}$ so $6 = \sqrt{a^2 - 64}$ and $a = 10$. The ellipse is given by $\frac{x^2}{10^2} + \frac{y^2}{8^2} = 1$.

3. A parabola with focus at (0,5) and vertex at (0,0). The parabola has an equation of the form $y = ax^2$ with $\frac{1}{4a} = 5$ so $a = \frac{1}{20}$. The parabola is given by $y = \frac{1}{20} x^2$.



Prob. 3

5. $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

7. Linear asymptotes: $y = \frac{2}{3} x$ and $y = -\frac{2}{3} x$. (Set $4x^2 - 9y^2 = 0$ and solve for y.) $c^2 = a^2 + b^2 = 4 + 9$ so $c = \sqrt{13}$: foci are at $(\sqrt{13}, 0)$ and $(-\sqrt{13}, 0)$.

9. Linear asymptotes: $y = \sqrt{\frac{3}{5}}\, x$ and $y = -\sqrt{\frac{3}{5}}\, x$. Foci: $(0, \sqrt{8})$ and $(0, -\sqrt{8})$.

11. (a) $25x^2 + 4y^2 + (-100) = 0$: discriminant $= (0)^2 - 4(25)(4) = -400 < 0$. The graph is an ellipse.
    (b) $b^2x^2 + a^2y^2 + (a^2b^2) = 0$: discriminant $= (0)^2 - 4(a^2)(b^2) = -4a^2b^2 < 0$. The graph is an ellipse.

13. $x^2 + xy - 2y^2 - x + y - 3 = 0$: discriminant $= (1)^2 - 4(1)(-2) = 9 > 0$. The graph is a hyperbola.

15. $2x^2 + 4xy + 2y^2 - 7x + 3 = 0$: discriminant $= (4)^2 - 4(2)(2) = 0$. The graph is a parabola.

17. (a) $B^2 - 4AC = 9 - 4(2)(2) < 0$: ellipse.
    (b) $B^2 - 4AC = 16 - 4(2)(2) = 0$: parabola.
    (c) $B^2 - 4AC = 25 - 4(2)(2) > 0$: hyperbola.
    (d) same answers as for parts (a), (b), and (c).

19. (a) $B^2 - 4AC = 16 - 4(1)(3) > 0$: hyperbola.
    (b) $B^2 - 4AC = 16 - 4(1)(4) = 0$: parabola.
    (c) $B^2 - 4AC = 16 - 4(1)(5) < 0$: ellipse.
    (d) same answers as for parts (a), (b), and (c).

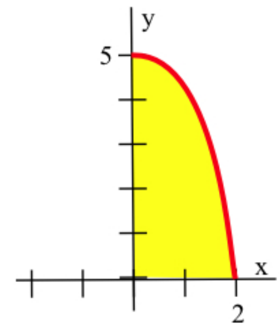
21. Figure is shown.

(a) About the x–axis: $V = 2\int_0^2 \pi y^2 \, dx = 2\pi \int_0^2 25\left(1 - \frac{x^2}{4}\right) dx$

$= 50\pi \left\{ x - \frac{x^3}{12} \right\} \Big|_0^2 = \frac{200\pi}{3}$.

(b) About the y–axis: $V = 2\int_0^5 \pi x^2 \, dy$

$= 2\pi \int_0^5 4\left(1 - \frac{y^2}{25}\right) dx = 8\pi \left\{ y - \frac{y^3}{75} \right\} \Big|_0^5 = \frac{80\pi}{3}$.



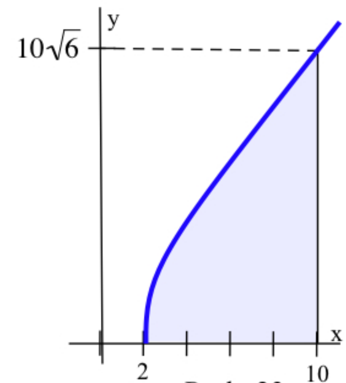
Prob. 21

23. Figure is shown.

(a) About the x–axis: $V = \int_2^{10} \pi y^2 \, dx$

$= \pi \int_2^{10} 25\left(\frac{x^2}{4} - 1\right) dx = 25\pi \left\{ \frac{x^3}{12} - x \right\} \Big|_2^{10}$

$= 25\pi\left\{ \left(\frac{1000}{12} - 10\right) - \left(\frac{8}{12} - 2\right) \right\} = \frac{5600\pi}{3} \approx 5864.3$.



Prob. 23

(b) About the y–axis: $V = 2\pi \int_0^{10\sqrt{6}} \left\{ 100 - 4\left(\frac{y^2}{25} + 1\right) \right\} dy = 2\pi \left\{ 96y - \frac{4y^3}{75} \right\} \Big|_0^{10\sqrt{6}}$

$= 20\sqrt{6}\,\pi \left\{ 96 - \frac{2400}{75} \right\} = \frac{3840\sqrt{6}\,\pi}{3} \approx 9850$.

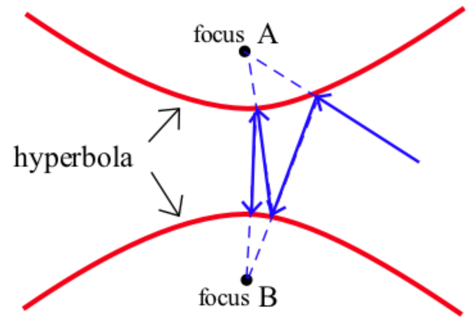
25. $A_{parabolic} = \int_a^b ax^2 \, dx = \frac{1}{3} ax^3 \Big|_a^b = \frac{1}{3} a b^3$. $A_{rectangular} = ab^2 \bullet b = a b^3$. $\frac{A_{parabolic}}{A_{rectangular}} = \frac{1}{3}$.

27. The length of the string is the distance between the vertices.

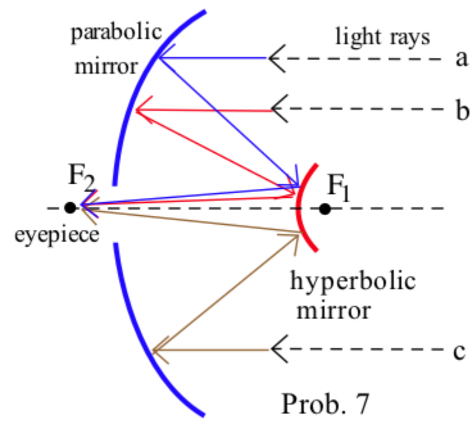
29. When the pins (foci) are far apart, the ellipse tends to be long and narrow, cigar shaped. As the pins are moved closer together, the ellipse becomes more rounded and circular. In the limit, with the pins together, the ellipse is a cirle with diameter equal to the length of the string.

31. The curves are parabolas As the string is shortened, the vertex is moved up and the parabola narrows, approaching a vertical ray in the limit as the length of the string nears the vertical distance from the pin to the corner of the T–square where the string is attached.

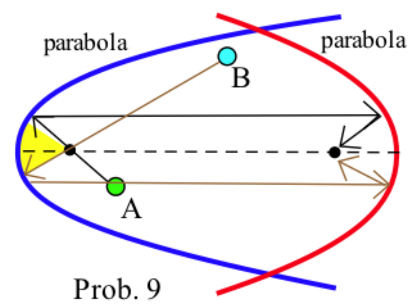
Section 9.6 Odd Answers



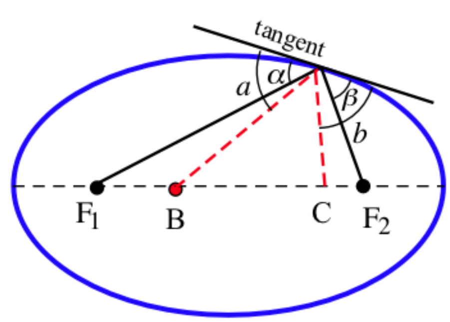
Prob. 3



Prob. 7



Prob. 9



Prob. 11

1. 12 units, independent of where it bounces off of the ellipse.

3. After a "long time," the ball oscillates (almost) along a line between the vertices of the hyperbola.

5. All of the energy along the wave front is reflected to the focus of the parabolic jetty at the same time The boat would be struck by a wave of considerable force.

7. The traced rays are shown. The eyepiece is located at F_2 because all of the incoming light is focused there.

9. (a) Roll the ball toward the focus F1 . The paths of A and B are shown.
   (b) The strategy in part (a) does not work for a ball in the shaded region. Why not?

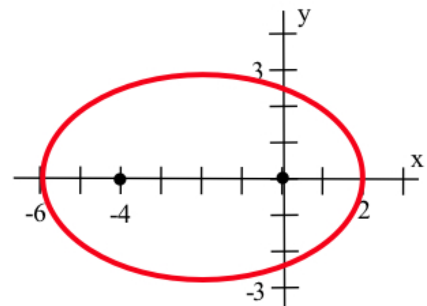
11. At any point E on the ellipse, the angle of incidence equals the angle of reflection so the angles α and β are equal and the angle a and b are equal. Since {angle a} > {angle α} we have that {angle b} > {angle β} and the ball is reflected to a point C between the two foci.

13. $r = \dfrac{11/3}{1 + (5/3)\cos(\theta)}$ so $e = 5/3 > 1$ and the graph is a hyperbola. The hyperbola crosses the x–axis when $\theta = 0$ and π: at the points $(11/8, 0)$ and $(11/2, 0)$. It crosses the y–axis when $\theta = \pi/2$ and $3\pi/2$: at the points $(0, 11/3)$ and $(0, -11/3)$.

15. $r = \dfrac{1/2}{1 + 1 \cdot \sin(\theta - \pi/3)}$ so $e = 1$ and the graph is a parabola.
    The parabola crosses the x–axis when $\theta = 0$ and π : approximately
    at the points $(3.73, 0)$ and $(-0.27, 0)$. It crosses the y–axis when $\theta = \pi/2$ and $3\pi/2$: at the points $(0, 1/3)$ and $(0, -1)$.

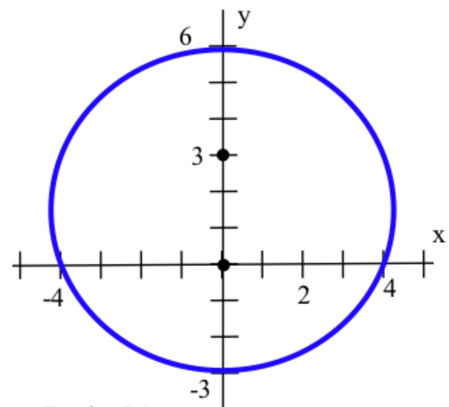
17. $r = \dfrac{17/7}{1-(5/7)\cdot\cos(\theta+3\pi)}$ so $e = 5/7 \le 1$ and the graph is an ellipse.

The ellipse crosses the x–axis when $\theta = 0$ and π: approximately at the points $(1.42, 0)$ and $(-8.5, 0)$. It crosses the y–axis when $\theta = \pi/2$ and $3\pi/2$: at the points $(0, 2.43)$ and $(0, -2.43)$.



Prob. 19

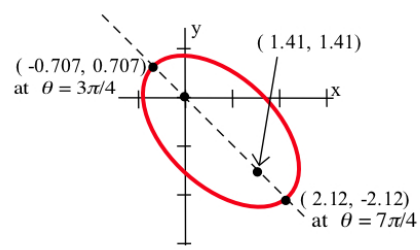
19. One focus is at (0,0) and, by symmetry, the other focus is at $(-4, 0)$.



Prob. 21

21. One focus is at (0,0) and, by symmetry, the other focus is at $(0, 3)$.

23. See Fig. 41. This one is more difficult because the ellipse is tilted, but we can still use the symmetry of the ellipse and the fact that it is tilted at an angle of $\pi/4$ to the x–axis. One focus is at (0,0) and the other focus is at $(1.41, -1.41)$.



Prob. 23

25. $r = \dfrac{1}{1+0.5\cdot\cos(\theta)}$.

$\dfrac{dr}{d\theta} = -(1+0.5\cdot\cos(\theta))^{-2}\cdot\{-0.5\cdot\sin(\theta)\} = \dfrac{\sin(\theta)}{2+\cos(\theta)}$.

$\text{Length} = \displaystyle\int_0^{2\pi} \sqrt{(r)^2 + (dr/d\theta)^2}\; d\theta \approx 7.659$

$\text{Area} = \displaystyle\int_0^{2\pi} \frac{1}{2}\, r^2(\theta)\, d \approx 4.8368$

Both integrals were approximated using Simpson's rule with $n = 20$)

27. $h = \dfrac{r_o v_o^2}{GM} = \dfrac{10^5\,(17.6)^2}{(6.7)(10^{-11})(10^{19})} = \dfrac{3.097 \times 10^7}{6.7 \times 10^8} \approx 4.62 \times 10^{-2}$ so $e = |h-1| \approx 0.95 < 1$

and the path is an ellipse (but a long narrow ellipse such as a comet might have).

29. $h = \dfrac{r_o v_o^2}{GM} = \dfrac{10^5\,(120)^2}{(6.7)(10^{-11})(10^{19})} = \dfrac{1.44 \times 10^9}{6.7 \times 10^8} \approx 2.149$ so $e = |h-1| \approx 1.149 > 1$ and

the path is a hyperbola.

31. $e = \frac{r \cdot v^2}{GM} - 1$.

(a) The path is circular if $e = 0$, so $\frac{r \cdot v^2}{GM} - 1 = 0$ and $v = \sqrt{\frac{GM}{r}}$.

(b) The path is elliptical if $e < 1$, so $\frac{r \cdot v^2}{GM} - 1 < 1$ and

$$v < \sqrt{\frac{2GM}{r}} = \sqrt{2}\sqrt{\frac{GM}{r}} = \sqrt{2} \cdot \{\text{ circular velocity}\}.$$

(c) The path is parabolic if $e = 1$, so $v = \sqrt{\frac{2GM}{r}} = \sqrt{2}\sqrt{\frac{GM}{r}} = \sqrt{2} \cdot \{\text{ circular velocity}\}$.

(d) The path is hyperbolic if $e > 1$, so $v > \sqrt{\frac{2GM}{r}} = \sqrt{2}\sqrt{\frac{GM}{r}} = \sqrt{2} \cdot \{\text{ circular velocity}\}$.

33. We want $v = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2(6.7 \text{ x } 10^{-11})(5.98 \text{ x } 10^{24})}{6.36 \text{ x } 10^{6}}} \approx \sqrt{1.26 \text{ x } 10^{8}} \approx 11{,}225$ m/s or

approximately 25,110 miles per hour.

35. $r_{max} = 1000 + r_{earth} \approx 7.36 \text{ x } 10^6$. $r_{min} = 800 + r_{earth} \approx 7.16 \text{ x } 10^6$.
$2a = r_{min} + r_{max} \approx 14.520 \text{ x } 10^6$ so $a \approx 7.26 \text{ x } 10^6$.
Also, $r_{max} = a(1 + e)$ so $7.36 \text{ x } 10^6 = 7.26 \text{ x } 10^6 (1 + e)$ and $e \approx 0.01377$.
Finally, $k = a(1 - e^2) \approx 7.26 \text{ x } 10^6 (1 - (0.01377)^2) \approx 7.2586 \text{ x } 10^6$ so

$$r = \frac{k}{1 + e \cdot \cos(\theta)} \approx \frac{7.2586 \text{ x } 10^6}{1 + (0.01377) \cdot \cos(\theta)} .$$

37. On your own.