I
Integral Table

To save space (and ink), only one member of each antiderivative family appears for most integrals below; for example, you should interpret $\int \cos(x)\,dx = \sin(x)$ as $\int \cos(x)\,dx = \sin(x) + C$, where C is an arbitrary constant.

Basic Patterns

$$\int k \cdot f(x)\,dx = k \cdot \int f(x)\,dx \qquad \int F'(ax+b)\,dx) = \frac{1}{a} \cdot F(x) \qquad \int F'(g(x)) \cdot g'(x)\,dx) = F(g(x))$$

$$\int [f(x) \pm g(x)]\ dx = \int f(x)\,dx \pm \int g(x)\,dx \qquad \int [f(x) \cdot g'(x)]\ dx = f(x) \cdot g(x) - \int f'(x) \cdot g(x)\,dx$$

Powers

$$\int x^p\,dx = \frac{1}{p+1}x^{p+1} \qquad (p \neq -1) \qquad \int \frac{1}{x}\,dx = \ln|x|$$

Exponential Functions

$$\int e^x\,dx = e^x \qquad \int a^x\,dx = \frac{a^x}{\ln(a)} \qquad (a > 0) \qquad \int x \cdot e^x\,dx = (x-1)e^x$$

$$\int x^2 \cdot e^x\,dx = (x^2 - 2x + 2)e^x \qquad \int x^n \cdot e^x\,dx = x^n \cdot e^x - n\int x^{n-1}e^x\,dx$$

Logarithmic Functions

$$\int \ln(x)\,dx = x\ln(x) - x \qquad \int \ln(a^2+x^2)\,dx = x \cdot \ln(a^2+x^2) + 2a\arctan\left(\frac{x}{a}\right) - 2x$$

$$\int \frac{\ln(x)}{x}\,dx = \frac{1}{2}\left[\ln(x)\right]^2 \qquad \int x^p \cdot \ln(x)\,dx = x^{p+1}\left[\frac{\ln(x)}{p+1} - \frac{1}{(p+1)^2}\right] \qquad (p \neq -1)$$

Trigonometric Functions

$$\int \sin(x)\,dx = -\cos(x)$$

$$\int \tan(x)\,dx = \ln\left(|\sec(x)|\right)$$

$$\int \sec(x)\,dx = \ln\left(|\sec(x)+\tan(x)|\right)$$

$$\int \cos(x)\,dx = \sin(x)$$

$$\int \cot(x)\,dx = \ln\left(|\sin(x)|\right)$$

$$\int \csc(x)\,dx = \ln\left(|\csc(x)-\cot(x)|\right)$$

$$\int \sin^2(x)\,dx = \frac{1}{2}x - \frac{1}{4}\sin(2x)$$

$$\int \sin^n(x)\,dx = -\frac{1}{n}\sin^{n-1}(x)\cos(x) + \frac{n-1}{n}\int \sin^{n-2}(x)\,dx$$

$$\int \cos^2(x)\,dx = \frac{1}{2}x + \frac{1}{4}\sin(2x)$$

$$\int \cos^n(x)\,dx = \frac{1}{n}\cos^{n-1}(x)\sin(x) + \frac{n-1}{n}\int \cos^{n-2}(x)\,dx$$

$$\int \sec^2(x)\,dx = \tan(x)$$

$$\int \sec^3(x)\,dx = \frac{1}{2}\sec(x)\tan(x) + \frac{1}{2}\ln\left(|\sec(x)+\tan(x)|\right)$$

$$\int \sec(x)\tan(x)\,dx = \sec(x)$$

$$\int \sec^n(x)\,dx = \frac{1}{n-1}\sec^{n-2}(x)\tan(x) + \frac{n-2}{n-1}\int \sec^{n-2}(x)\,dx$$

$$\int \csc^2(x)\,dx = -\cot(x)$$

$$\int \csc^3(x)\,dx = -\frac{1}{2}\csc(x)\cot(x) + \frac{1}{2}\ln\left(|\csc(x)-\cot(x)|\right)$$

$$\int \csc(x)\cot(x)\,dx = -\csc(x)$$

$$\int \csc^n(x)\,dx = -\frac{1}{n-1}\csc^{n-2}(x)\cot(x) + \frac{n-2}{n-1}\int \csc^{n-2}(x)\,dx$$

$$\int \sin(ax)\cos(bx)\,dx = -\frac{\cos\left((a-b)x\right)}{2(a-b)} - \frac{\cos\left((a+b)x\right)}{2(a+b)} \quad (a \neq \pm b)$$

$$\int \sin(ax)\sin(bx)\,dx = \frac{\sin\left((a-b)x\right)}{2(a-b)} - \frac{\sin\left((a+b)x\right)}{2(a+b)} \quad (a \neq \pm b)$$

$$\int x\sin(x)\,dx = -x\cos(x) + \sin(x)$$

$$\int \cos(ax)\cos(bx)\,dx = \frac{\sin\left((a-b)x\right)}{2(a-b)} + \frac{\sin\left((a+b)x\right)}{2(a+b)} \quad (a \neq \pm b)$$

$$\int x\cos(x)\,dx = x\sin(x) + \cos(x)$$

$$\int x^n\sin(x)\,dx = -x^n\cos(x) + n\int x^{n-1}\cos(x)\,dx$$

$$\int x^n\cos(x)\,dx = x^n\sin(x) - n\int x^{n-1}\sin(x)\,dx$$

Hyperbolic Functions

$$\int \sinh(x)\,dx = \cosh(x)$$

$$\int \tanh(x)\,dx = \ln\left(\cosh(x)\right)$$

$$\int \operatorname{sech}(x)\,dx = \arctan(\sinh(x))$$

$$\int \cosh(x)\,dx = \sinh(x)$$

$$\int \coth(x)\,dx = \ln\left(|\sinh(x)|\right)$$

$$\int \operatorname{csch}(x)\,dx = \ln\left(|\coth(x)-\operatorname{csch}(x)|\right)$$

$$\int \operatorname{sech}^2(x)\,dx = \tanh(x)$$

$$\int \operatorname{sech}(x)\tanh(x)\,dx = -\operatorname{sech}(x)$$

$$\int \operatorname{csch}^2(x)\,dx = -\coth(x)$$

$$\int \operatorname{csch}(x)\coth(x)\,dx = -\operatorname{csch}(x)$$

Inverse Trigonometric Functions

$$\int \arcsin(x)\,dx = x \cdot \arcsin(x) + \sqrt{1-x^2}$$

$$\int \arctan(x)\,dx = x \cdot \arctan(x) - \frac{1}{2}\ln(1+x^2)$$

Rational Functions

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right)$$

$$\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\ln\left(\left|\frac{x+a}{x-a}\right|\right) = \frac{1}{a}\operatorname{argtanh}\left(\frac{x}{a}\right)$$

$$\int \frac{1}{(a^2+x^2)^2}\,dx = \frac{1}{2a^3}\left[\frac{ax}{a^2+x^2} + \arctan\left(\frac{x}{a}\right)\right]$$

$$\int \frac{1}{(x-a)(x-b)}\,dx = \frac{1}{a-b}\ln\left(\left|\frac{x-a}{x-b}\right|\right)$$

Radical Functions

$$\int \sqrt{x^2 \pm a^2}\,dx = \frac{x}{2}\sqrt{x^2 \pm a^2} + \frac{a^2}{2}\ln\left(x + \sqrt{x^2 \pm a^2}\right)$$

$$\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

$$\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln\left(x + \sqrt{x^2 \pm a^2}\right)$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \arcsin\left(\frac{x}{a}\right)$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \operatorname{argsinh}\left(\frac{x}{a}\right)$$

$$\int \frac{1}{x\sqrt{x^2 - a^2}}\,dx = \frac{1}{a}\operatorname{arcsec}\left(\frac{x}{a}\right) \quad (x > a)$$

Products of Exponentials and Trigonometric or Hyperbolic Functions

$$\int e^{ax}\sin(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin(bx) - b\cos(bx)\right]$$

$$\int e^{ax}\cos(bx)\,dx = \frac{e^{ax}}{a^2+b^2}\left[a\cos(bx) + b\sin(bx)\right]$$

$$\int e^{ax}\sinh(bx)\,dx = \frac{e^{ax}}{a^2-b^2}\left[a\sinh(bx) - b\cosh(bx)\right]$$

$$\int e^{ax}\cosh(bx)\,dx = \frac{e^{ax}}{a^2-b^2}\left[a\cosh(bx) - b\sinh(bx)\right]$$