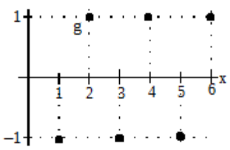
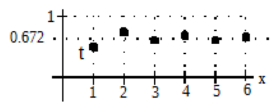
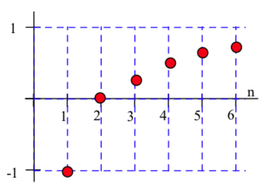
Section 9.1

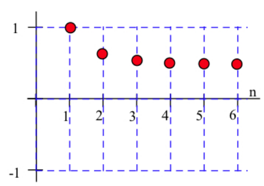
1. (a) 32, 64 (b) $a_5 = 2^5$ (c) $a_n = 2^n$
3. (a) -1, 1 (b) $a_5 = (-1)^5$ (c) $a_n = (-1)^n$
5. (a) 120, 720 (b) $a_5 = 5!$ (c) $a_n = n!$
7. $1, \frac{3}{2}, \frac{11}{6}, \frac{25}{12}, \frac{137}{60}, \frac{49}{20}$
9. $1, \frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{11}{16}, \frac{21}{32}$
11. 1, 0, 1, 0, 1, 0
13. (a) $g(5) = -1$, $g(6) = 1$ (b) See below left.



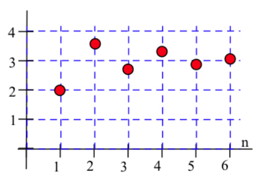


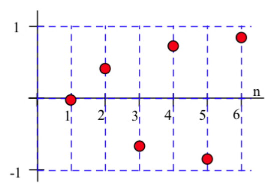
15. (a) $t(5) = \frac{21}{32}$, $t(6) = \frac{43}{64}$ (b) See above right.
17. $a_n = \frac{1}{n^2}$
19. $a_n = \frac{n-1}{n}$
21. $a_n = \frac{n}{2^n}$
23. $-1, 0, \frac{1}{3}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}$; see below left.



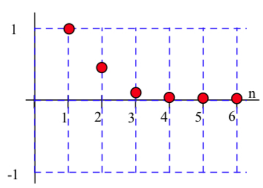


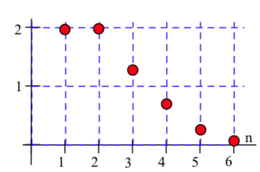
25. $1, \frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{5}{9}, \frac{6}{11}$; see above right.
27. $2, \frac{7}{2}, \frac{8}{3}, \frac{13}{4}, \frac{14}{5}, \frac{19}{6}$; see below left.





29. $0, \frac{1}{2}, -\frac{2}{3}, \frac{3}{4}, -\frac{4}{5}, \frac{5}{6}$; see above right.
31. $1, \frac{1}{2}, \frac{1}{6}, \frac{1}{24}, \frac{1}{120}, \frac{1}{720}$; see below left.





33. $2, 2, \frac{4}{3}, \frac{2}{3}, \frac{4}{15}, \frac{4}{45}$; see above right.
35. 2, -2, 2, -2, 2, -2, 2, -2, 2, -2
37. $\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, 0, \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, 0, \frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}, 0, \frac{\sqrt{3}}{2}$
39. 1, 3, 6, 10, 15, 21, 28, 36, 45, 55

Section 9.2

1. $\{a_n\}$ appears to converge; $\{b_n\}$ does not.
3. $\{f_n\}$ appears to converge; $\{e_n\}$ does not.
5. converges to 1
7. diverges (to ∞)
9. converges to $\frac{1}{2}$
11. converges to $\ln(3)$
13. diverges
15. converges to 0
17. converges to $e^{-1} = \frac{1}{e}$
19. converges to 1
21. Given $\epsilon > 0$, take $N \geq \frac{\sqrt{3}}{\sqrt{\epsilon}}$. Then:

$$n > N \Rightarrow n > \frac{\sqrt{3}}{\sqrt{\epsilon}} \Rightarrow n^2 > \frac{3}{\epsilon} \Rightarrow \frac{3}{n^2} < \epsilon$$

so that $|a_n - L| = \left|\frac{3}{n^2} - 0\right| = \frac{3}{n^2} < \epsilon$.

23. Given $\epsilon > 0$, take $N \geq \frac{1}{\epsilon}$. Then:

$$n > N \Rightarrow n > \frac{1}{\epsilon} \Rightarrow \frac{1}{n} < \epsilon$$

so that $|a_n - L| = \left|\frac{3n-1}{n} - 3\right| = \frac{1}{n} < \epsilon$.

25. The given sequence is a subsequence of $\left\{\frac{1}{n}\right\}$, which converges (to 0), so the given subsequence must also converge (to 0).
27. The sequence simplifies to $\{(-1)^n\}$: the subsequence of even-indexed terms is $\{1, 1, 1, \ldots\}$, which converges to 1, while the subsequence of odd-indexed terms is $\{-1, -1, -1, \ldots\}$, which converges to -1. Because the given sequence has two subsequences that converge to different limits, the original sequence diverges.
29. The given sequence is a subsequence of $\left\{\left(1 + \frac{5}{k}\right)^k\right\}$, which converges to e^5, so the given sequence also converges to e^5.
31. $a_{n+1} - a_n = \left[7 - \frac{2}{n+1}\right] - \left[7 - \frac{2}{n}\right] = \frac{2}{n} - \frac{2}{n+1} = \frac{2}{n(n+1)} > 0$ so $\{a_n\}$ is monotonically increasing.
33. $a_{n+1} - a_n = 2^{n+1} - 2^n == 2^n\,[2 - 1] = 2^n > 0$ so the sequence $\{a_n\}$ is monotonically increasing.
35. $a_{n+1} - a_n = \left[5 + \frac{7}{3^{n+1}}\right] - \left[5 + \frac{7}{3^n}\right] = \frac{7}{3^{n+1}} - \frac{7}{3^n} = \frac{-14}{3^{n+1}} < 0$ so $\{a_n\}$ is monotonically decreasing.

37. $a_n = \frac{n+1}{n!} \Rightarrow a_{n+1} = \frac{n+2}{(n+1)!} \Rightarrow \frac{a_{n+1}}{a_n} = \frac{n+2}{(n+1)!} \cdot \frac{n!}{n+1} = \frac{(n+2)\cdot n!}{(n+1)^2 \cdot n!} = \frac{n+2}{(n+1)^2} < 1$ when $n \geq 1$, so $\{a_n\}$ is monotonically decreasing.

39. $a_n = \left(\frac{5}{4}\right)^n \Rightarrow a_{n+1} = \left(\frac{5}{4}\right)^{n+1} \Rightarrow \frac{a_{n+1}}{a_n} = \frac{5}{4} > 1$, so $\{a_n\}$ is monotonically increasing.

41. $a_n = \frac{n}{e^n} \Rightarrow a_{n+1} = \frac{n+1}{e^{n+1}} \Rightarrow \frac{a_{n+1}}{a_n} = \frac{n+1}{e^{n+1}} \cdot \frac{e^n}{n} = \frac{(n+1)}{n \cdot e} < 1$ when $n \geq 1$, so $\{a_n\}$ is monotonically decreasing.

43. $f(x) = 5 - \frac{3}{x} \Rightarrow f'(x) = \frac{3}{x^2} > 0$, so $f(x)$ is always increasing, which means that $\left\{5 - \frac{3}{n}\right\}$ is monotonically increasing.

45. $f(x) = \cos\left(\frac{1}{x}\right) \Rightarrow f'(x) = \frac{1}{x^2}\sin\left(\frac{1}{x}\right)$, so $f'(x) > 0$ for $x \geq 1$, meaning that $f(x)$ is increasing and $\left\{\cos\left(\frac{1}{n}\right)\right\}$ is monotonically increasing.

47. $a_n = \frac{n+3}{n!} \Rightarrow a_{n+1} = \frac{n+4}{(n+1)!} \Rightarrow \frac{a_{n+1}}{a_n} = \frac{n+4}{(n+1)!} \cdot \frac{n!}{n+3} = \frac{(n+4)\cdot n!}{(n+1)(n+3)\cdot n!} = \frac{n+4}{(n+1)(n+3)} < 1$ when $n \geq 1$, so $\{a_n\}$ is monotonically decreasing.

49. $a_{n+1} - a_n = \left[1 - \frac{1}{2^{n+1}}\right] - \left[1 - \frac{1}{2^n}\right] = \frac{1}{2^n} - \frac{1}{2^{n+1}} = \frac{1}{2^{n+1}} > 0$ so the sequence $\left\{1 - \frac{1}{2^n}\right\}$ is monotonically increasing.

51. $a_n = \frac{n+1}{e^n} \Rightarrow a_{n+1} = \frac{n+2}{e^{n+1}} \Rightarrow \frac{a_{n+1}}{a_n} = \frac{n+2}{e^{n+1}} \cdot \frac{e^n}{n+1} = \frac{(n+2)}{(n+1)\cdot e} =< 1$ when $n \geq 1$, so $\left\{\frac{n+1}{e^n}\right\}$ is monotonically decreasing.

53. For $N = 4$: $a_1 = 4 \Rightarrow a_2 = \frac{1}{2}\left(4 + \frac{4}{4}\right) = 2.5 \Rightarrow a_3 = \frac{1}{2}\left(2.5 + \frac{4}{2.5}\right) = 2.05 \Rightarrow a_4 = \frac{1}{2}\left(2.05 + \frac{4}{2.05}\right) \approx 2.00061$. For $N = 9$: $a_1 = 9 \Rightarrow a_2 = \frac{1}{2}\left(9 + \frac{9}{9}\right) = 5 \Rightarrow a_3 = \frac{1}{2}\left(5 + \frac{9}{5}\right) = 3.2 \Rightarrow a_4 = \frac{1}{2}\left(3.2 + \frac{9}{3.2}\right) = 3.00625$. For $N = 5$: $a_1 = 5 \Rightarrow a_2 = \frac{1}{2}\left(5 + \frac{5}{5}\right) = 3 \Rightarrow a_3 = \frac{1}{2}\left(3 + \frac{5}{3}\right) \approx 2.333 \Rightarrow a_4 \approx \frac{1}{2}\left(2.333 + \frac{5}{2.333}\right) \approx 2.238$.

55. (a) Solving $0.01 = \frac{0.02}{0.02k+1}$ for k:

$$0.02k + 1 = \frac{0.02}{0.01} = 2 \Rightarrow 0.02k = 1 \Rightarrow k = \frac{1}{0.02}$$

or 50 generations.

(b) Solving $\frac{1}{2}p = \frac{p}{kp+1}$ for k in terms of p:

$$kp + 1 = \frac{p}{0.5p} = 2 \Rightarrow kp = 1 \Rightarrow k = \frac{1}{p}$$

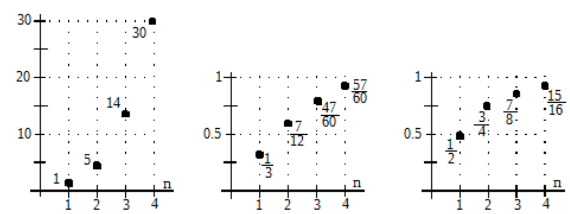
57. (a) The first "few" grains can be anywhere on the x-axis. (b) After placing "a lot of grains," there will be a large pile of sand close to $x = 3$.

59. (a) $-1 \leq \sin(n) \leq 1$ for all integers n, so the first few grains will be scattered between -1 and $+1$ on the x-axis. (b) After placing "a lot of grains," the sand will be scattered "uniformly" along the interval from -1 to $+1$. (c) A formal proof of this fact is rather sophisticated, but the result is interesting: no two grains ever end up on the same point on the x-axis.

Section 9.3

1. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k}$ 3. $\displaystyle\sum_{k=1}^{\infty} \frac{2}{3k}$ 5. $\displaystyle\sum_{k=1}^{\infty} \left(-\frac{1}{2}\right)^k$

7. $s_1 = 1$, $s_2 = 1 + 4 = 5$, $s_3 = 5 + 9 = 14$, $s_4 = 14 + 16 = 30$; see below left.



9. $s_1 = \frac{1}{3}$, $s_2 = \frac{7}{12}$, $s_3 = \frac{47}{60}$, $s_4 = \frac{19}{20}$; above center.

11. $s_1 = \frac{1}{2}$, $s_2 = \frac{3}{4}$, $s_3 = \frac{7}{8}$, $s_4 = \frac{15}{16}$; above right.

13. $a_1 = s_1 = 3$, $a_2 = s_2 - s_1 = 2 - 3 = -1$, $a_3 = s_3 - s_2 = 4 - 2 = 2$, $a_4 = s_4 - s_3 = 5 - 4 = 1$

15. $a_1 = 4$, $a_2 = 0.5$, $a_3 = -0.2$, $a_4 = 0.5$

17. $a_1 = 1$, $a_2 = 0.1$, $a_3 = 0.01$, $a_4 = 0.001$

19. $0.888\ldots = 0.8 + 0.08 + 0.008 + \cdots = \displaystyle\sum_{k=1}^{\infty} \frac{8}{10^k}$

21. $\displaystyle\sum_{k=1}^{\infty} \frac{5}{10^k}$ 23. $\displaystyle\sum_{k=1}^{\infty} \frac{a}{10^k}$ 25. $\displaystyle\sum_{k=1}^{\infty} \frac{17}{100^k}$

27. $\displaystyle\sum_{k=1}^{\infty} \frac{7}{100^k}$ 29. $\displaystyle\sum_{k=1}^{\infty} \frac{abc}{1000^k}$ 31. $\displaystyle\sum_{k=0}^{\infty} 30\,(0.8)^k$

33. 80%, 64%, 51.2%, $(0.8)^n \cdot 100\%$

35. $\displaystyle\lim_{k\to\infty} \left(\frac{1}{4}\right)^k = 0$, so $\displaystyle\sum_{k=1}^{\infty} \left(\frac{1}{4}\right)^k = 0$ may or may not converge. (Section 9.4 shows it converges.)

37. $\lim_{k\to\infty} \left(\frac{4}{3}\right)^k = \infty \neq 0$, so $\sum_{k=1}^{\infty} \left(\frac{4}{3}\right)^k$ diverges.

39. $\lim_{k\to\infty} \frac{\sin(k)}{k} = 0$, so $\sum_{k=1}^{\infty} \frac{\sin(k)}{k}$ may or may not converge. (Techniques you may learn in more advanced courses will show that it converges.)

41. $\lim_{k\to\infty} \cos(k)$ does not exist, so $\sum_{k=1}^{\infty} \cos(k)$ diverges.

43. $\lim_{k\to\infty} \frac{k^2-20}{k^5+4} = 0$, so $\sum_{k=1}^{\infty} \frac{k^2-20}{k^5+4}$ may or may not converge. (We'll see later that it converges.)

45. Let $s_n = \sum_{k=1}^{n} a_k$. Then:

$$cA = c \cdot \sum_{k=1}^{\infty} a_k = c \cdot \lim_{n\to\infty} s_n = \lim_{n\to\infty} c \cdot s_n$$
$$= \lim_{n\to\infty} c \cdot \sum_{k=1}^{n} a_k = \lim_{n\to\infty} \sum_{k=1}^{n} c \cdot a_k = \sum_{k=1}^{\infty} c \cdot a_k$$

47. If $s_n = \sum_{k=1}^{n} a_k$, $\sum_{k=1}^{\infty} a_k = A$ means that $\lim_{n\to\infty} s_n = A$. We also know that $\lim_{n\to\infty} s_{n-1} = A$ and that $a_n = s_n - s_{n-1}$, so:

$$\lim_{n\to\infty}, a_n = \lim_{n\to\infty}, [s_n - s_{n-1}]$$
$$= \lim_{n\to\infty}, s_n - \lim_{n\to\infty}, s_{n-1} = A - A = 0$$

Section 9.4

1. This is a geometric series with $|r| = \frac{2}{7} < 1$, so it converges to:

$$\frac{1}{1-r} = \frac{1}{1-\frac{2}{7}} = \frac{1}{\frac{5}{7}} = \frac{7}{5}$$

3. This is a geometric series with $|r| = \frac{4}{7} < 1$, so it converges to:

$$\frac{1}{1-r} = \frac{1}{1-\left(-\frac{4}{7}\right)} = \frac{1}{\frac{11}{7}} = \frac{7}{11}$$

5. This is a geometric series with $|r| = \frac{2}{7} < 1$, so it converges, but the index starts at $k = 1$, hence:

$$\sum_{k=1}^{\infty} \left(\frac{2}{7}\right)^k = \left[\sum_{k=0}^{\infty} \left(\frac{2}{7}\right)^k\right] - \left(\frac{2}{7}\right)^0$$
$$= \frac{1}{1-\frac{2}{7}} - 1 = \frac{7}{5} - 1 = \frac{2}{5}$$

7. This geometric series diverges: $|r| = \frac{7}{4} > 1$.

9. This is a geometric series with $|r| = \frac{2}{7} < 1$, so it converges, but the index starts at $k = 5$, hence:

$$\left[\sum_{k=0}^{\infty} \left(\frac{2}{7}\right)^k\right] - \left[\sum_{k=0}^{4} \left(-\frac{2}{7}\right)^k\right]$$
$$= \frac{1}{1+\frac{2}{7}} - \left[1 - \frac{2}{7} + \frac{4}{49} - \frac{8}{343} + \frac{16}{2401}\right]$$
$$= -\frac{32}{21609} \approx -0.00148$$

11. This geometric series diverges: $|r| = \frac{\pi}{3} > 1$.

13. $\sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k = \frac{1}{1-\frac{1}{3}} = \frac{3}{2}$

15. $\sum_{k=3}^{\infty} \left(\frac{1}{2}\right)^k = \frac{1}{8} \cdot \sum_{k=3}^{\infty} \left(\frac{1}{2}\right)^k = \frac{1}{8} \cdot \frac{1}{1-\frac{1}{2}} = \frac{1}{4}$

17. $\sum_{k=1}^{\infty} \left(-\frac{2}{3}\right)^k = -\frac{2}{3} \cdot \sum_{k=0}^{\infty} \left(-\frac{2}{3}\right)^k = -\frac{2}{3} \cdot \frac{3}{5} = -\frac{2}{5}$

19. (a) $\frac{1}{1-\frac{1}{2}} - 1 = \frac{1}{\frac{1}{2}} - 1 = 2 - 1 = 1$

(b) $\frac{1}{1-\frac{1}{3}} - 1 = \frac{1}{\frac{2}{3}} - 1 = \frac{3}{2} - 1 = \frac{1}{2}$

(c) $\frac{1}{1-\frac{1}{a}} - 1 = \frac{1}{\frac{a-1}{a}} - 1 = \frac{a}{a-1} - \frac{a-1}{a-1} = \frac{1}{a-1}$

21. (a) $40(0.4)^n$ (b) $\sum_{n=0}^{\infty} 40(0.4)^n$ (c) $\frac{40}{1-0.4} \approx 66.67$ ft

23. (a) $\sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n$ (b) $\frac{1}{2}, \frac{1}{4}, \left(\frac{1}{2}\right)^n$ (c) All of it.

25. $\sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k = \frac{1}{1-\frac{1}{4}} = \frac{4}{3}$

27. (a) We can express the total area as:

$$1 + 3 \cdot \frac{1}{9} + 3 \cdot 4 \cdot \frac{1}{9^2} + 3 \cdot 4^2 \cdot \frac{1}{9^3} + \cdots$$
$$= 1 + \frac{3}{9} + \frac{3}{9} \cdot \frac{4}{9} + + \frac{3}{9} \cdot \frac{4^2}{9^2} + \cdots$$
$$= 1 + \frac{1}{3}\left[1 + \frac{4}{9} + \left(\frac{4}{9}\right)^2 + \cdots\right]$$
$$= 1 + \frac{1}{3} \cdot \frac{1}{1-\frac{4}{9}} = 1 + \frac{1}{3} \cdot \frac{9}{5} = 1 + \frac{3}{5} = 1.6$$

(b) Let L be the length of one side of the original triangle, so the perimeter of the original triangle is $3L$. The first step replaces each original side with four smaller sides each $\frac{1}{3}$ the length of the original side, so the perimeter after the first step is $3 \cdot 4 \cdot \frac{1}{3}L = 4L$. The second step replaces each of the 12 existing sides with four smaller sides each $\frac{1}{3}$ the length of the previous sides, so the perimeter is now $3 \cdot 4^2 \cdot \frac{1}{3}\left(\frac{1}{3}L\right) = 3L\left(\frac{4}{3}\right)^2$. In general, the perimeter after n steps is $3L\left(\frac{4}{3}\right)^n$, which has limit ∞ as n increases without bound.

29. (a) The total height can be expressed as:

$$2 \cdot 1 + 2 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} + 2 \cdot \frac{1}{8} + \cdots = \frac{2}{1 - \frac{1}{2}} = 4$$

(b) The total surface area is:

$$\begin{aligned} &4\pi \cdot 1^2 + 4\pi\left(\frac{1}{2}\right)^2 + 4\pi\left(\frac{1}{4}\right)^2 + 4\pi\left(\frac{1}{8}\right)^2 + \cdots \\ &= 4\pi\left[1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \cdots\right] \\ &= 4\pi \cdot \frac{1}{1 - \frac{1}{4}} = 4\pi \cdot \frac{4}{3} = \frac{16\pi}{3} \approx 16.755 \end{aligned}$$

(c) The total volume is:

$$\begin{aligned} &\frac{4}{3}\pi \cdot 1^3 + \frac{4}{3}\pi \cdot \left(\frac{1}{2}\right)^3 + \frac{4}{3}\pi \cdot \left(\frac{1}{4}\right)^3 + \cdots \\ &= \frac{4}{3}\pi\left[1 + \frac{1}{8} + \left(\frac{1}{8}\right)^2 + \cdots\right] \\ &= \frac{4}{3}\pi \cdot \frac{1}{1 - \frac{1}{8}} = \frac{4}{3}\pi \cdot \frac{8}{7} = \frac{32\pi}{21} \approx 4.787 \end{aligned}$$

31. We can rewrite $0.\overline{8} = 0.888\ldots$ as:

$$\frac{8}{10} \cdot \sum_{k=0}^{\infty} \left(\frac{1}{10}\right)^k = \frac{8}{10} \cdot \frac{1}{\frac{9}{10}} = \frac{8}{9}$$

Similarly, $0.\overline{9} = 0.999\ldots = \frac{9}{10} \cdot \frac{1}{\frac{9}{10}} = 1$ and $0.\overline{285714} = \frac{285714}{1000000} \cdot \frac{1}{\frac{999999}{1000000}} = \frac{285714}{999999}$.

33. The series converges precisely when:

$$\begin{aligned} &|2x+1| < 1 \Rightarrow -1 < 2x+1 < 1 \\ &\Rightarrow -2 < 2x < 0 \Rightarrow -1 < x < 0 \end{aligned}$$

35. The series converges if and only if:

$$\begin{aligned} &|1-2x| < 1 \Rightarrow -1 < 2x-1 < 1 \\ &\Rightarrow 0 < 2x < 2 \Rightarrow 0 < x < 1 \end{aligned}$$

37. The series converges when x satisfies:

$$|7x| < 1 \Rightarrow -1 < 7x < 1 \Rightarrow -\frac{1}{7} < x < \frac{1}{7}$$

39. The ratio is $\frac{x}{2}$, so the series converges when:

$$\left|\frac{x}{2}\right| < 1 \Rightarrow -1 < \frac{x}{2} < 1 \Rightarrow -2 < x < 2$$

41. The ratio is $2x$, so the series converges when:

$$|2x| < 1 \Rightarrow -1 < 2x < 1 \Rightarrow -\frac{1}{2} < x < \frac{1}{2}$$

43. The ratio is $\sin(x)$, so the series converges when: $|\sin(x)| < 1$, which holds true for all values of x except odd multiples of $\frac{\pi}{2}$.

45. The first student stated the formula correctly, but it is valid only when $|x| < 1$, so the second student should not have put $x = 2$ into the formula.

47. This is a telescoping sum: $s_4 = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$, $s_5 = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$, $s_n = \frac{1}{3} - \frac{1}{n+1}$

49. This is a telescoping sum: $s_4 = 1^3 - 5^3 = -124$, $s_5 = 1^3 - 6^3 = -215$, $s_n = 1^3 - (n+1)^3$

51. This is a telescoping sum: $s_4 = f(1) - f(5)$, $s_5 = f(1) - f(6)$, $s_n = f(1) - f(n+1)$

53. $s_4 = \sin(1) - \sin\left(\frac{1}{5}\right) \approx 0.643$
$s_5 = \sin(1) - \sin\left(\frac{1}{6}\right) \approx 0.676$
$s_n = \sin(1) - \sin\left(\frac{1}{n+1}\right) \to \sin(1) \approx 0.841$

55. $s_4 = \frac{1}{4} - \frac{1}{25} = 0.21$; $s_5 = \frac{1}{4} - \frac{1}{36} = \frac{2}{9}$
$s_n = \frac{1}{4} - \frac{1}{(n+1)^2} \to \frac{1}{4}$

57. On your own.

59. On your own.

61. (a) $\frac{3}{4}$ (b) $\frac{c}{(c-2)^2}$

Section 9.5

1. Sum. 3. $\sum_{k=1}^{\infty} f(k)$ 5. $\sum_{k=2}^{\infty} f(k)$

7. $f(1) + f(2)$ 9. $f(2) + f(3)$

11. $f(1) + f(2) + f(3) < \int_1^4 f(x)\,dx$
$< f(2) + f(3) + f(4) < \int_2^5 f(x)\,dx$

13. (a) You did well. (b) You may have done well or you may have done poorly. (c) You may have done well or poorly. (d) You did poorly.

15. (a) Unknown is good. (b) Unknown might be good or might be bad. (c) Unknown might be good or might be bad. (d) Unknown is bad.

17. $\sum_{k=2} \frac{1}{k^3-5}$ 19. $\sum_{k=1} \frac{1}{k^2+5k}$

21. (a) $k+4$ (b) $\frac{k+4}{k+3}$ (c) $\frac{k+4}{k+3}$

23. (a) $\frac{3}{k+1}$ (b) $\frac{\frac{3}{k}}{\frac{3}{k+1}}$ (c) $\frac{k}{k+1}$

25. (a) 2^{k+1} (b) $\frac{2^{k+1}}{2^k}$ (c) 2 27. (a) x^{k+1} (b) x (c) x

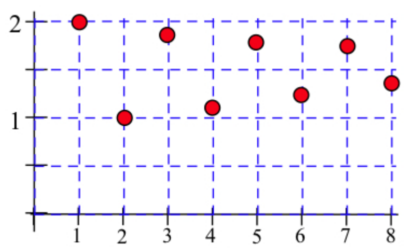
29. converges; $\frac{1}{2}$ 31. diverges; 2

33. diverges; 1 35. diverges; $\frac{k}{k+1}$

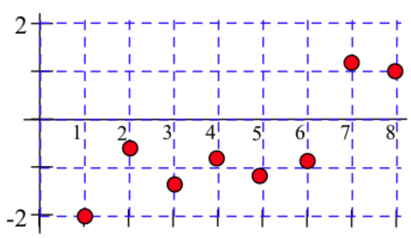
37. $<$ 39. $>$ 41. $>$

43. $s_3 < s_5 < s_4 < s_6$ 45. $s_5 < s_6 < s_4 < s_3$

47. $s_1 = 2$, $s_2 = 1$, $s_3 = 1.9$, $s_4 = 1.1$, $s_5 = 1.8$, $s_6 = 1.2$, $s_7 = 1.7$, $s_8 = 1.3$; "funnel-shaped":



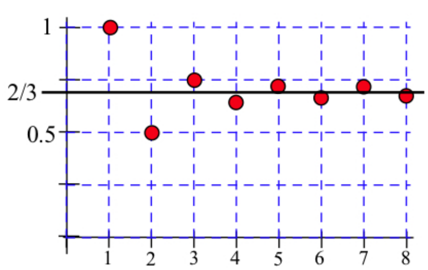
49. $s_1 = -2$, $s_2 = -0.5$, $s_3 = -1.3$, $s_4 = -0.7$, $s_5 = -1.1$, $s_6 = -0.9$, $s_7 = 1.1$, $s_8 = 1.0$; initially "funnel-shaped":



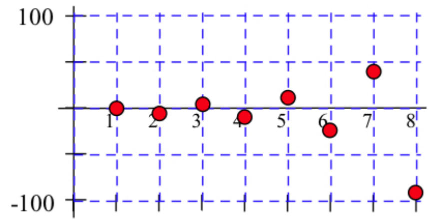
51. The terms a_k need to alternate in sign.

53. (a) D, E, F (b) A, D (c) D

55.



57.



Section 9.6

1. $f(x) = (2x+5)^{-1}$ is positive, continuous and decreasing on $[1, \infty)$, and:

$$\lim_{M\to\infty} \int_1^M \frac{1}{2x+5}\,dx = \lim_{M\to\infty} \left[\frac{\ln(2x+5)}{2}\right]_1^M = \infty$$

so $\int_1^\infty \frac{1}{2x+5}\,dx$ and $\sum_{k=1}^\infty \frac{1}{2k+5}$ both diverge.

3. $f(x) = (2x+5)^{-\frac{3}{2}}$ is positive, continuous and decreasing on $[1, \infty)$, and:

$$\int_1^M (2x+5)^{-\frac{3}{2}}\,dx = \left[\frac{-1}{\sqrt{2x+5}}\right]_1^M \to \frac{1}{\sqrt{7}}$$

as $M \to \infty$ so $\int_1^\infty (2x+5)^{-\frac{3}{2}}\,dx$ converges, hence $\sum_{k=1}^\infty \frac{1}{(2k+5)^{\frac{3}{2}}}$ converges.

5. On $[2, \infty)$, $f(x) = \frac{1}{x \cdot [\ln(x)]^2}$ is positive, continuous and decreasing, and:

$$\int_1^M \frac{1}{1 \cdot [\ln(x)]^2}\,dx = \left[\frac{-1}{\ln(x)}\right]_1^M \to \frac{1}{\ln(2)}$$

as $M \to \infty$ so $\int_1^\infty \frac{1}{1 \cdot [\ln(x)]^2}\,dx$ converges, hence $\sum_{k=2}^\infty \frac{1}{k \cdot [\ln(k)]^2}$ converges.

7. $f(x) = \frac{1}{1+x^2}$ is positive, continuous and decreasing (everywhere), and:

$$\int_1^M \frac{1}{1+x^2}\,dx = [\arctan(x)]_1^M \to \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

as $M \to \infty$ so $\int_1^M \frac{1}{1+x^2}\,dx$ converges, hence $\sum_{k=1}^\infty \frac{1}{k^2+1}$ converges.

9. $\displaystyle\sum_{k=1}^{\infty}\left[\frac{1}{k}-\frac{1}{k+3}\right]$ is a telescoping series:

$$\left[1-\frac{1}{4}\right]+\left[\frac{1}{2}-\frac{1}{5}\right]+\left[\frac{1}{3}-\frac{1}{6}\right]+\left[\frac{1}{4}-\frac{1}{7}\right]+\left[\frac{1}{5}-\frac{1}{8}\right]+\cdots=1+\frac{1}{2}+\frac{1}{3}$$

so the series converges to $\frac{11}{6}$. The Integral Test also works because $f(x)=\dfrac{1}{x}-\dfrac{1}{x+3}$ is positive, continuous and decreasing (you should verify this), and:

$$\lim_{M\to\infty}\int_1^M\left[\frac{1}{x}-\frac{1}{x+3}\right]dx=\lim_{M\to\infty}\Big[\ln(x)-\ln(x+3)\Big]_1^M=\lim_{M\to\infty}\left[\ln\left(\frac{M}{M+3}\right)-\ln(1)+\ln(4)\right]=\ln(4)$$

Because the improper integral converges, the series converges as well, but the Integral Test does not tell us the sum of the series. In this instance, the "telescoping series" method is both easier and more precise.

11. Applying the Integral Test to $\displaystyle\sum_{k=1}^{\infty}\frac{1}{k(k+5)}$ using $f(x)=\dfrac{1}{x(x+5)}$ (you should verify that this function is positive, continuous and decreasing) and Partial Fraction Decomposition:

$$\lim_{M\to\infty}\int_1^M\frac{1}{5}\left[\frac{1}{x}-\frac{1}{x+5}\right]dx=\lim_{M\to\infty}\frac{1}{5}\Big[\ln(x)-\ln(x+5)\Big]_1^M=\lim_{M\to\infty}\frac{1}{5}\left[\ln\left(\frac{M}{M+5}\right)+\ln(6)\right]=\frac{\ln(6)}{5}$$

Because the improper integral converges, the series converges as well, but we can use the same partial fraction decomposition to turn the series into a telescoping series and find its exact value:

$$\left[1-\frac{1}{6}\right]+\left[\frac{1}{2}-\frac{1}{7}\right]+\left[\frac{1}{3}-\frac{1}{8}\right]+\left[\frac{1}{4}-\frac{1}{9}\right]+\left[\frac{1}{5}-\frac{1}{10}\right]+\left[\frac{1}{6}-\frac{1}{11}\right]+\cdots=1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}=\frac{137}{60}$$

13. Applying the Integral Test to $\displaystyle\sum_{k=1}^{\infty}ke^{-k^2}$ using $f(x)=xe^{-x^2}$ (verify it is positive, continuous and decreasing):

$$\lim_{M\to\infty}\int_1^M xe^{-x^2}\,dx=\lim_{M\to\infty}\left[\frac{-1}{2e^{x^2}}\right]_1^M=\lim_{M\to\infty}\left[\frac{-1}{2e^{M^2}}+\frac{1}{2e}\right]=\frac{1}{2e}$$

Because the improper integral converges, the series converges as well.

15. Applying the Integral Test to $\displaystyle\sum_{k=1}^{\infty}\frac{1}{\sqrt{6k+10}}$ using $f(x)=\dfrac{1}{\sqrt{6x+10}}$ (verify it is positive, continuous and decreasing):

$$\lim_{M\to\infty}\int_1^M(6x+10)^{-\frac{1}{2}}\,dx=\lim_{M\to\infty}\left[\frac{1}{3}\sqrt{6x+10}\right]_1^M=\lim_{M\to\infty}\frac{1}{3}\left[\sqrt{6M+10}-4\right]=\infty$$

Because the improper integral diverges, the series diverges as well.

17. converges ($p=4>1$)

19. diverges ($p=\frac{1}{5}\le 1$)

21. diverges ($p=1\le 1$)

23. converges ($p=\frac{3}{2}>1$)

25. converges ($p=\frac{4}{3}>1$)

27. diverges ($p=\frac{2}{3}\le 1$)

29. $\displaystyle\int_1^{11}\frac{1}{x^3}\,dx\le s_{10}\le 1+\int_1^{10}\frac{1}{x^3}\,dx\ \Rightarrow\ 0.4958677<s_{10}<1.495$

$\displaystyle\int_1^{101}\frac{1}{x^3}\,dx\le s_{100}\le 1+\int_1^{100}\frac{1}{x^3}\,dx\ \Rightarrow\ 0.0.499951<s_{100}<1.49995$

$\displaystyle\int_1^{1000001}\frac{1}{x^3}\,dx\le s_{1000000}\le 1+\int_1^{1000000}\frac{1}{x^3}\,dx\ \Rightarrow\ 0.5000000<s_{1000000}<1.5000000$

31. $\ln(11) < s_{10} < 1 + \ln(10)$,
$4.6151 < s_{100} < 5.6052$,
$13.8155 < s_{1000000} < 14.8155$

33. $\arctan(11) - \frac{\pi}{4} < s_{10} < \frac{1}{2} + \arctan(11) - \frac{\pi}{4}$,
$0.7755 < s_{100} < 1.2754$,
$0.7854 < s_{1000000} < 1.2854$

35. $s_{10} = \sum_{k=1}^{10} \frac{1}{k^4} \approx 1.08203658 \Rightarrow s_{11} \approx 1.08210488$ and $\int_{11}^{\infty} \frac{1}{x^4}\, dx = \frac{1}{3993} \approx 0.00025044$, so:

$$1.08228702 < \sum_{k=1}^{\infty} \frac{1}{k^4} < 1.08235532$$

Using $n = 20$ yields:

$$1.08232058 < \sum_{k=1}^{\infty} \frac{1}{k^4} < 1.08232572$$

37. $\int_{11}^{\infty} \frac{1}{x^2+1}\, dx \approx 0.09065989$, $s_{10} \approx 0.98179282$ and $s_{11} \approx 0.98998954$, so:

$$1.07245271 < \sum_{k=1}^{\infty} \frac{1}{k^2+1} < 1.08064943$$

Using $n = 20$ yields:

$$1.07552492 < \sum_{k=1}^{\infty} \frac{1}{k^2+1} < 1.07778736$$

39. Using $n = 10$: $2.5984 < \sum_{k=1}^{\infty} \frac{1}{k\sqrt{k}} < 2.6258$
Using $n = 20$: $2.6071 < \sum_{k=1}^{\infty} \frac{1}{k\sqrt{k}} < 2.6175$

41. Use the substitution $u = \ln(x) \Rightarrow du = \frac{1}{x}\, dx$ and the Integral Test to see that the improper integral:

$$\int_{2}^{\infty} \frac{1}{x \cdot [\ln(x)]^q}\, dx$$

diverges for $q \leq 1$ and converges for $q > 1$, hence the series does as well.

43. converges ($q = 3 > 1$)

45. diverges ($\ln\left(k^3\right) = 3\ln(k)$ so $q = 1 \leq 1$)

Section 9.7

1. $0 \leq \cos^2(k) \leq 1 \Rightarrow 0 \leq \frac{\cos^2(k)}{k^2} \leq \frac{1}{k^2}$ so $\sum_{k=1}^{\infty} \frac{\cos^2(k)}{k^2}$ converges by BCT with $\sum_{k=1}^{\infty} \frac{1}{k^2}$.

3. $n - 1 < n \Rightarrow \frac{5}{n-1} \geq \frac{5}{n}$ so $\sum_{n=3}^{\infty} \frac{5}{n-1}$ diverges by comparison with $\sum_{n=3}^{\infty} \frac{5}{n}$, which diverges because it is a multiple of p-series with $p = 1$.

5. $3 + \cos(m) \geq 2$ so $\sum_{m=1}^{\infty} \frac{3+\cos(m)}{m}$ diverges by comparison with $\sum_{m=1}^{\infty} \frac{2}{m}$, which diverges because it is a multiple of the harmonic series.

7. For $k \geq 3$, $\ln(k) > 1$ and $\frac{\ln(k)}{k} > \frac{1}{k}$, so $\sum_{k=2}^{\infty} \frac{\ln(k)}{k}$ diverges by comparison with $\sum_{k=2}^{\infty} \frac{1}{k}$.

9. $k \geq 9 \Rightarrow 0 < \frac{k+9}{k \cdot 2^k} \leq \frac{2k}{k \cdot 2^k} = \frac{1}{2^{k-1}}$ so $\sum_{k=1}^{\infty} \frac{k+9}{k \cdot 2^k}$ converges by comparison with $\sum_{k=1}^{\infty} \frac{1}{2^{k-1}}$ (a geometric series with ratio $\frac{1}{2} < 1$).

11. $k \geq 2 \Rightarrow k! \geq k(k-1) \geq k\left(k - \frac{1}{2}k\right) = \frac{1}{2}k^2 \Rightarrow 0 < \frac{1}{k!} < \frac{2}{k^2}$ so $\sum_{k=1}^{\infty} \frac{1}{k!}$ converges by comparison with $\sum_{k=1}^{\infty} \frac{2}{k^2}$ (a p-series with $p = 2 > 1$).

13. Using the LCT with the harmonic series:

$$\lim_{k\to\infty} \frac{\frac{k+1}{k^2+4}}{\frac{1}{k}} = \lim_{k\to\infty} \frac{k^2+k}{k^2+4} = 1$$

so $\sum_{k=3}^{\infty} \frac{k+1}{k^2+4}$ because $\sum_{k=3}^{\infty} \frac{1}{k}$ diverges.

15. Diverges by LCT with the harmonic series.

17. Converges by LCT with the p-series $\sum_{k=1}^{\infty} \frac{1}{k^3}$:

$$\lim_{k\to\infty} \frac{\frac{k^3}{(1+k^2)^3}}{\frac{1}{k^3}} = \lim_{k\to\infty} \frac{k^6}{k^6+2k^4+3k^2+1} = 1$$

19. Diverges by LCT with harmonic series.

21. Converges by LCT with the p-series $\sum_{k=1}^{\infty} \frac{1}{k^3}$

23. Converges by LCT with the p-series $\sum_{n=3}^{\infty} \frac{1}{n^2}$

25. Diverges by LCT with the p-series $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$

27. Converges by LCT with the p-series $\sum_{k=2}^{\infty} \frac{1}{k^3}$

29. Converges by LCT with the p-series $\sum_{k=1}^{\infty} \frac{1}{k^2}$

31. Diverges by LCT with the p-series $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$

33. Diverges by LCT with harmonic series.

35. Diverges by LCT with harmonic series.

37. Diverges by LCT with harmonic series.

39. Diverges by LCT with the p-series $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$

41. Converges by LCT with $\sum_{k=1}^{\infty} \frac{1}{3^k}$

43. Diverges by Test for Divergence.

45. Converges by LCT with $\sum_{k=1}^{\infty} \frac{1}{e^k}$

47. Converges by LCT with the p-series $\sum_{n=1}^{\infty} \frac{1}{n^2}$

49. Diverges by LCT with harmonic series.

51. Converges by LCT with the p-series $\sum_{n=1}^{\infty} \frac{1}{n^3}$

53. Diverges by Test for Divergence.

55. Diverges by Test for Divergence.

57. Converges: geometric series with $r = \frac{1}{3}$.

59. Diverges by Test for Divergence.

61. Converges: geometric series with $r = e^{-1}$.

63. Converges: geometric series with $r = \frac{\pi^2}{e^3} < \frac{1}{2}$.

65. Converges by BCT with the p-series $\sum_{n=1}^{\infty} \frac{1}{n^3}$

67. Converges by Integral Test.

69. Diverges by LCT with harmonic series.

71. For $x \geq 5$, $\ln(x) < \sqrt{x}$; to verify this, note that:

$$\mathbf{D}\left(\ln(x)\right) = \frac{1}{x} \leq \frac{1}{2\sqrt{x}} = \mathbf{D}\left(\sqrt{x}\right)$$

and $\ln(5) < \sqrt{5}$. Hence for $k \geq 5$:

$$\frac{\ln(k)}{k^2} < \frac{\sqrt{k}}{k^2} = \frac{1}{k^{\frac{3}{2}}}$$

so $\sum_{k=2}^{\infty} \frac{\ln(k)}{k^2}$ converges by BCT with $\sum_{k=2}^{\infty} \frac{1}{k^{\frac{3}{2}}}$.

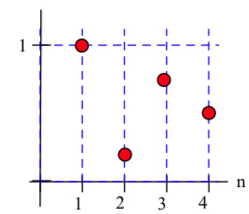
73. Diverges by LCT with harmonic series.

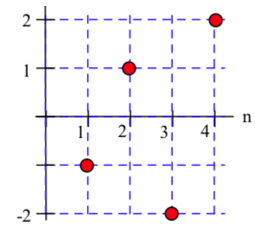
75. Diverges by LCT with harmonic series.

77. Diverges by Test for Divergence.

Section 9.8

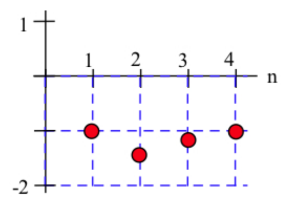
1. (a) See below left. (b) Alternating (so far).





3. (a) See above right. (b) Alternating (so far).

5. (a) See below. (b) Not alternating.



7. Alternating: $a_1 = 2$, $a_2 = -1$, $a_3 = 2$, $a_4 = -1$, $a_5 = 2$

9. Not alternating: $a_1 = 2$, $a_2 = 1$, $a_3 = -0.9$, $a_4 = 0.8$, $a_5 = -0.1$

11. Not alternating: $a_1 = -1$, $a_2 = 2$, $a_3 = -1.2$, $a_4 = 0.2$, $a_5 = 0.2$

13. A has decreasing partial sums, so terms are all negative; C has increasing partial sums, so terms are all positive.

15. B has increasing partial sums, so terms are all positive (they do not alternate).

17. Converges by AST. 19. Converges by AST.

21. Diverges by Test for Divergence.

23. Converges by AST. 25. Converges by AST.

27. Diverges by Test for Divergence.

29. The AST does *not* apply to this series because the terms are all negative, but it does converge; factor out -2 and use the LCT with the resulting series and a geometric series.

31. Converges because all terms are 0.

33. (a) $s_4 = 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} = \frac{115}{144} \approx 0.79861$
(b) $|a_5| = \frac{1}{25} = 0.04$ (c) $0.75861 < S < 0.83861$

35. (a) $s_4 = \frac{1}{\ln(2)} - \frac{1}{\ln(3)} + \frac{1}{\ln(4)} - \frac{1}{\ln(5)} \approx 0.6325$
(b) $|a_5| = \frac{1}{\ln(6)} \approx 0.5581$ (c) $0.0744 < S < 1.1906$

37. (a) $s_4 \approx 0.20992$ (b) $|a_5| = (0.8)^6 \approx 0.26214$
(c) $-0.05222 < S < 0.47206$

39. (a) $s_4 \approx 0.441836$ (b) $|a_5| = \sin\left(\frac{1}{5}\right) \approx 0.198669$
(c) $0.243167 < S < 0.640505$

41. (a) $s_4 = -1 + \frac{1}{8} - \frac{1}{27} + \frac{1}{64} \approx -0.896412$
(b) $|a_5| = 0.008$ (c) $-0.904412 < S < -0.888412$

43. $\frac{1}{(N+1)+6} \le \frac{1}{100} \Rightarrow N + 7 \ge 100 \Rightarrow N \ge 93$

45. We need $\frac{2}{\sqrt{(N+1)+21}} \le \frac{1}{100} \Rightarrow \sqrt{N+22} \ge 200 \Rightarrow N + 22 \ge 40000 \Rightarrow N \ge 39978$

47. $\left(\frac{1}{3}\right)^{N+1} \le \frac{1}{500} \Rightarrow N + 1 \ge \frac{\ln(500)}{\ln(3)} \approx 5.66$, so use $N = 5$.

49. We need $\frac{1}{(N+1)^4} \le \frac{1}{1000} \Rightarrow (N+1)^4 \ge 1000 \Rightarrow N + 1 > 5.62 \Rightarrow N > 4.62$, so use $N = 5$.

51. $\frac{1}{(N+1)+\ln(N+1)} \le \frac{1}{25} \Rightarrow (N+1) + \ln(N+1) \ge 25$; this will certainly be true if $N+1 \ge 25 \Rightarrow N \ge 24$ (but some experimenting with a calculator shows that $N = 21$ works while $N = 20$ does not).

53. (a) $S(0.3) = 0.3 - \dfrac{(0.3)^3}{3!} + \dfrac{(0.3)^5}{5!} - \dfrac{(0.3)^7}{7!} + \cdots$
(b) $s_3 = 0.3 - \frac{(0.3)^3}{3!} + \frac{(0.3)^5}{5!} \approx 0.29552025$
(c) $|S - s_3| \le \frac{(0.3)^7}{7!} \approx 0.000000043$

55. (a) $S(0.1) = 0.1 - \dfrac{(0.1)^3}{3!} + \dfrac{(0.1)^5}{5!} - \dfrac{(0.1)^7}{7!} + \cdots$
(b) $s_3 = 0.1 - \frac{(0.1)^3}{3!} + \frac{(0.1)^5}{5!} \approx 0.09983342$
(c) $|S - s_3| \le \frac{(0.1)^7}{7!} \approx 2 \times 10^{-11}$

57. (a) $C(1) = 1 - \dfrac{1}{2!} + \dfrac{1}{4!} - \dfrac{1}{6!} + \cdots$
(b) $s_3 = 1 - \frac{1}{2!} + \frac{1}{4!} \approx 0.5416667$
(c) $|S - s_3| \le \frac{1}{6!} \approx 0.0013889$

59. (a) $1 - \dfrac{(-0.2)^2}{2!} + \dfrac{(-0.2)^4}{4!} - \dfrac{(-0.2)^6}{6!} + \cdots$
(b) $s_3 = 1 - \frac{(-0.2)^2}{2!} + \frac{(-0.2)^4}{4!} \approx 0.980066667$
(c) $|S - s_3| \le \frac{(-0.2)^6}{6!} \approx 9 \times 10^{-8}$

61. (a) $1 + (-1) + \dfrac{(-1)^2}{2!} + \dfrac{(-1)^3}{3!} + \dfrac{(-1)^4}{4!} + \cdots$
(b) $s_3 = 1 - 1 + \frac{1}{2} = 0.5$
(c) $|S - s_3| \le \frac{1}{3!} = \frac{1}{6} \approx 0.16667$

63. (a) $1 + (-0.2) + \dfrac{(-0.2)^2}{2!} + \dfrac{(-0.2)^3}{3!} + \dfrac{(-0.2)^4}{4!} + \cdots$
(b) $s_3 = 1 - 0.2 + \frac{0.04}{2} = 0.82$
(c) $|S - s_3| \le \frac{(0.2)^4}{6} \approx 0.0013333$

Section 9.9

1. $\displaystyle\sum_{k=1}^{\infty} \left|\frac{(-1)^{k+1}}{k+2}\right| = \sum_{k=1}^{\infty} \frac{1}{k+2}$, which diverges (by LCT with the harmonic series) so $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k+2}$ is not absolutely convergent, however the AST applies, so it converges conditionally.

3. $\displaystyle\sum_{n=1}^{\infty} \left|(-1)^{n+1} \cdot \frac{5}{n^3}\right| = 5\sum_{n=1}^{\infty} \frac{1}{n^3}$, which converges, so $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \cdot \frac{5}{n^3}$ converges absolutely.

5. $\displaystyle\sum_{k=0}^{\infty} \left|(-0.5)^k\right| = \sum_{k=0}^{\infty} (0.5)^k$ is a convergent geometric series, so $\displaystyle\sum_{k=0}^{\infty} (-0.5)^k$ converges absolutely.

7. $\displaystyle\sum_{k=1}^{\infty} \left|\frac{(-1)^{k+1}}{k^2}\right| = \sum_{k=1}^{\infty} \frac{1}{k^2}$ converges (by the P-Test), so $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k^2}$ converges absolutely.

9. $\displaystyle\sum_{n=2}^{\infty} \left|(-1)^n \cdot \frac{\ln(n)}{n}\right| = \sum_{n=2}^{\infty} \frac{\ln(n)}{n}$ diverges (by BCT with the harmonic series), but the AST says $\displaystyle\sum_{n=2}^{\infty} (-1)^n \cdot \frac{\ln(n)}{n}$ converges conditionally.

11. $\displaystyle\sum_{k=1}^{\infty} \left|\frac{(-1)^k}{k+\ln(k)}\right| = \sum_{k=1}^{\infty} \frac{1}{k+\ln(k)}$ diverges by BCT with $\displaystyle\sum_{k=1}^{\infty} \frac{1}{2k}$, but the AST applies to $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{k+\ln(k)}$, so it converges conditionally.

13. $\displaystyle\sum_{k=1}^{\infty} \left|(-1)^k \cdot \sin\left(\frac{1}{k}\right)\right| = \sum_{k=1}^{\infty} \sin\left(\frac{1}{k}\right)$ diverges (by LCT with the harmonic series), but the AST says $\displaystyle\sum_{k=1}^{\infty} (-1)^k \cdot \sin\left(\frac{1}{k}\right)$ converges conditionally.

15. $\sum_{k=1}^{\infty} \sqrt{k} \sin\left(\frac{1}{k^2}\right)$ converges by LCT with $\sum_{k=1}^{\infty} \frac{1}{k^{\frac{3}{2}}}$, so $\sum_{k=1}^{\infty} (-1)^k \sqrt{k} \sin\left(\frac{1}{k^2}\right)$ converges absolutely.

17. $\sum_{m=2}^{\infty} (-1)^m \cdot \frac{\ln(m)}{\ln(m^3)}$ diverges, because the terms do not approach 0.

19. Converges conditionally.

21. Diverges by Test for Divergence.

23. Converges absolutely (all terms are 0).

25. Converges conditionally.

27. Diverges by Test for Divergence.

29. Converges absolutely.

31. $\frac{n!}{(n+1)!} = \frac{n!}{(n+1) \cdot n!} = \frac{1}{n}$

33. $\frac{(n-1)!}{(n+1)!} = \frac{(n-1)!}{(n+1) \cdot n \cdot (n-1)!} = \frac{1}{n(n+1)}$

35. $\frac{n!}{(n+2)!} = \frac{n!}{(n+2)(n+1) \cdot n!} = \frac{1}{(n+1)(n+2)}$

37. $\frac{2 \cdot n!}{n! \cdot (n+1)(n+2) \cdots (2n)} = \frac{2}{(n+1)(n+2) \cdots (2n)}$

39. $\frac{n \cdot n \cdot n \cdots n \cdot n}{1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n} = \frac{n}{1} \cdot \frac{n}{2} \cdot \frac{n}{3} \cdots \frac{n}{n-1} \cdot \frac{n}{n}$

41. $\frac{\frac{1}{k+1}}{\frac{1}{k}} = \frac{k}{k+1} \to 1$, so the Ratio Test is inconclusive; series diverges (harmonic series).

43. $\frac{\frac{1}{(k+1)^3}}{\frac{1}{k^3}} = \left(\frac{k}{k+1}\right)^3 \to 1$, so the Ratio Test is inconclusive; series converges (P-Test).

45. $\frac{\left(\frac{1}{2}\right)^{k+1}}{\left(\frac{1}{2}\right)^{k}} = \frac{1}{2} < 1$; absolutely convergent (AC).

47. $\frac{1^{n+1}}{1^n} = 1$, so the Ratio Test is inconclusive; diverges (by Test for Divergence).

49. $\frac{\frac{1}{(k+1)!}}{\frac{1}{k!}} = \frac{k!}{(k+1)!} = \frac{1}{k+1} \to 0 < 1$; AC.

51. $\frac{\frac{2^{k+1}}{(k+1)!}}{\frac{2^k}{k!}} = 2 \cdot \frac{k!}{(k+1)!} = \frac{2}{k+1} \to 0 < 1$; AC.

53. $\frac{\left(\frac{1}{2}\right)^{3k+3}}{\left(\frac{1}{2}\right)^{3k}} = \frac{1}{8} < 1$; converges absolutely.

55. $\frac{(0.9)^{2k+3}}{(0.9)^{2k+1}} = 0.81 < 1$; converges absolutely.

57. $\left|\frac{(-1.1)^{k+1}}{(-1.1)^k}\right| = 1.1 > 1$; diverges.

59. $\left|\frac{(x-5)^{k+1}}{(x-5)^k}\right| = |x-5| < 1 \Rightarrow 4 < x < 6$. At $x = 4$ and $x = 6$ the series diverges (by the Test for Divergence), so the series converges absolutely on $(4, 6)$ and diverges elsewhere.

61. $\left|\frac{\frac{(x-5)^{k+1}}{(k+1)^2}}{\frac{(x-5)^k}{k^2}}\right| = \left(\frac{k}{k+1}\right)^2 |x-5| \to |x-5| < 1 \Rightarrow 4 < x < 6$. At $x = 4$ and $x = 6$ the series converges absolutely (by the P-Test), so the series converges absolutely on $[4, 6]$ and diverges elsewhere.

63. $\left|\frac{\frac{(x-2)^{k+1}}{(k+1)!}}{\frac{(x-2)^k}{k!}}\right| = \frac{1}{k+1} \cdot |x-2| \to 0 < 1$ for all x, so the series converges absolutely on $(-\infty, \infty)$.

65. Converges absolutely on $\left[\frac{11}{2}, \frac{13}{2}\right]$.

67. Converges absolutely on $(-\infty, \infty)$.

69. $\left|\frac{\frac{(x+1)^{2k+2}}{k+1}}{\frac{(x+1)^{2k}}{k}}\right| = \frac{k}{k+1} \cdot (x+1)^2 \to (x+1)^2 < 1 \Rightarrow |x+1| < 1 \Rightarrow -2 < x < 0$; at $x = -2$ the series diverges and at $x = 0$ the series diverges, so it converges absolutely on $(-2, 0)$.

71. Converges absolutely on $[4, 6]$.

73. Converges absolutely on $(-\infty, \infty)$.

75. Converges absolutely on $(-\infty, \infty)$.

77. Converges absolutely on $(-\infty, \infty)$.

79. $\sqrt[k]{\left(\frac{2}{7}\right)^k} = \frac{2}{7} < 1$; absolutely convergent.

81. $\sqrt[k]{\frac{1}{k^3}} \to 1$, so Root Test inconclusive; absolutely convergent by P-Test.

83. $\sqrt[k]{\frac{1}{k^k}} = \frac{1}{k} \to 0 < 1$; absolutely convergent.

85. $\sqrt[k]{\left(\frac{1}{2}-\frac{2}{k}\right)^k} = \frac{1}{2}-\frac{2}{k} \to \frac{1}{2} < 1$; AC.

87. $\sqrt[k]{\left(\frac{2+k}{k}\right)^k} = \frac{2+k}{k} \to 1$, so Root Test inconclusive; diverges by Test for Divergence.

89. $\sqrt[k]{|\cos(k\pi)|^k} = 1$, so Root Test is inconclusive; diverges by Test for Divergence.

91. $L = \frac{2}{3} < 1$; absolutely convergent.

93. $\sqrt[k]{\frac{(2k)^k}{k^{2k}}} = \frac{2k}{k^2} = \frac{2}{k} \to 0 < 1$; AC.

95. $\frac{1}{2} - 1 + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} + \frac{1}{12} + \frac{1}{14} + \frac{1}{16} - \frac{1}{3} + \frac{1}{18} + \frac{1}{20} + \frac{1}{22} + \frac{1}{24} + \frac{1}{26}$

97. $\frac{1}{\sqrt{2}} + \frac{1}{2} - 1 + \frac{1}{\sqrt{6}} + \frac{1}{\sqrt{8}} + \frac{1}{\sqrt{10}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{12}} + \frac{1}{\sqrt{14}} - \frac{1}{\sqrt{5}} + \frac{1}{4} - \frac{1}{\sqrt{7}} + \frac{1}{\sqrt{18}} + \frac{1}{\sqrt{20}} - \frac{1}{3}$

99. $\frac{1}{\sqrt{2}} - 1 + \frac{1}{2} + \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{8}} + \frac{1}{\sqrt{10}} - \frac{1}{\sqrt{5}} + \frac{1}{\sqrt{12}} - \frac{1}{\sqrt{7}} + \frac{1}{\sqrt{14}} - \frac{1}{3} + \frac{1}{4} + \frac{1}{\sqrt{18}} - \frac{1}{\sqrt{11}}$

101. On your own.

103. On your own.

Section 10.1

1. This is a geometric series with ratio x, so it converges precisely when $|x| < 1$; the interval of convergence is $(-1,1)$. (Graph it yourself.)

3. Applying the Ratio Test:
$$\left|\frac{3^{k+1}\cdot x^{k+1}}{3^k \cdot x^k}\right| = |3x|$$
for all values of x, so the series converges when $|3x| < 1 \Rightarrow |x| < \frac{1}{3}$ and diverges when $|x| > \frac{1}{3}$. At $x = \frac{1}{3}$ the series becomes $\sum_{k=1}^{\infty} 1$, which diverges by the Test for Divergence; at $x = -\frac{1}{3}$, the series becomes $\sum_{k=1}^{\infty} (-1)^k$, which also diverges by the Test for Divergence. The interval of convergence is therefore $(-\frac{1}{3}, \frac{1}{3})$. (The graph is left to you.)

5. Applying the Ratio Test:
$$\left|\frac{\frac{x^{k+1}}{k+1}}{\frac{x^k}{k}}\right| = \frac{k}{k+1}\cdot|x| \longrightarrow |x|$$
so the series converges when $|x| < 1$ and diverges when $|x| > 1$. At $x = 1$ the series becomes the harmonic series, which diverges; at $x = -1$, the series becomes the alternating harmonic series, which converges conditionally (by the Alternating Series Test). The interval of convergence is therefore $[-1, 1)$. (The graph is left to you.)

7. Applying the Ratio Test:
$$\left|\frac{(k+1)\cdot x^{k+1}}{k \cdot x^k}\right| = \frac{k+1}{k}\cdot|x| \longrightarrow |x|$$
so the series converges when $|x| < 1$ and diverges when $|x| > 1$. At $x = 1$ the series becomes $\sum_{k=1}^{\infty} k$, which diverges by the Test for Divergence; at $x = -1$, the series becomes $\sum_{k=1}^{\infty} k\cdot(-1)^k$, which also diverges by the Test for Divergence. The interval of convergence is therefore $(-1,1)$.

9. Applying the Ratio Test:
$$\left|\frac{(k+1)\cdot x^{2k+3}}{k \cdot x^{2k+1}}\right| = \frac{k+1}{k}\cdot x^2 \longrightarrow x^2$$
so the series converges when $x^2 < 1 \Rightarrow |x| < 1$ and diverges when $|x| > 1$. At $x = 1$ the series becomes $\sum_{k=1}^{\infty} k$, which diverges by the Test for Divergence; at $x = -1$, the series becomes $\sum_{k=1}^{\infty} -k$, which also diverges by the Test for Divergence. The interval of convergence is therefore $(-1,1)$.

11. Applying the Ratio Test:
$$\left|\frac{\frac{x^{k+1}}{(k+1)!}}{\frac{x^k}{k!}}\right| = \frac{k!\cdot|x|}{(k+1)!} = \frac{k!\cdot|x|}{(k+1)\cdot k!} = \frac{|x|}{k+1} \longrightarrow 0$$
for any x, so the interval of convergence is therefore $(-\infty, \infty)$.

13. Applying the Ratio Test:
$$\left|\frac{(k+1)\cdot\frac{x^{2k+2}}{4^{2k+2}}}{k\cdot\frac{x^{2k}}{4^{2k}}}\right| = \frac{(k+1)\cdot x^2}{16k} \longrightarrow \frac{x^2}{16}$$
so the series converges when $\frac{x^2}{16} < 1 \Rightarrow x^2 < 16 \Rightarrow |x| < 4$ and diverges when $|x| > 4$. At $x = \pm 4$ the series becomes $\sum_{k=1}^{\infty} k$, which diverges, so the interval of convergence is $(-4,4)$.